

Topic Y8 Functions and series (Pre-TT A) [45]

1.

11. The second, third and fourth terms of an arithmetic sequence are $2k$, $5k - 10$ and $7k - 14$ respectively, where k is a constant.

Show that the sum of the first n terms of the sequence is a square number.

(5)

(Total 5 marks)

2.

- (i) Solve the inequality $|2x + 1| \leq |x - 3|$.

[5]

- (ii) Given that x satisfies the inequality $|2x + 1| \leq |x - 3|$, find the greatest possible value of $|x + 2|$.

[2]

(Total 7 marks)

3.

10. The function f is defined by

$$f: x \mapsto \frac{3x - 5}{x + 1}, \quad x \in \mathbb{R}, \quad x \neq -1$$

- (a) Find $f^{-1}(x)$.

(3)

- (b) Show that

$$ff(x) = \frac{x + a}{x - 1}, \quad x \in \mathbb{R}, \quad x \neq \pm 1$$

where a is an integer to be found.

(4)

The function g is defined by

$$g: x \mapsto x^2 - 3x, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 5$$

- (c) Find the value of $fg(2)$.

(2)

- (d) Find the range of g .

(3)

- (e) Explain why the function g does not have an inverse.

(1)

(Total 13 marks)

4.

Sarah is carrying out a series of experiments which involve using increasing amounts of a chemical. In the first experiment she uses 6 g of the chemical and in the second experiment she uses 7.8 g of the chemical.

- (i) Given that the amounts of the chemical used form an arithmetic progression, find the total amount of chemical used in the first 30 experiments. [3]
- (ii) Instead it is given that the amounts of the chemical used form a geometric progression. Sarah has a total of 1800 g of the chemical available. Show that N , the greatest number of experiments possible, satisfies the inequality

$$1.3^N \leq 91,$$

and use logarithms to calculate the value of N .

[6]

(Total 9 marks)

5.

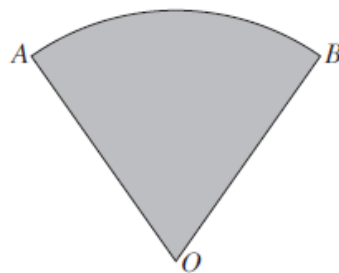


Fig. 1

Fig. 1 shows a sector AOB of a circle, centre O and radius OA . The angle AOB is 1.2 radians and the area of the sector is 60 cm^2 .

- (i) Find the perimeter of the sector.

[4]

A pattern on a T-shirt, the start of which is shown in Fig. 2, consists of a sequence of similar sectors. The first sector in the pattern is sector AOB from Fig. 1, and the area of each successive sector is $\frac{3}{5}$ of the area of the previous one.

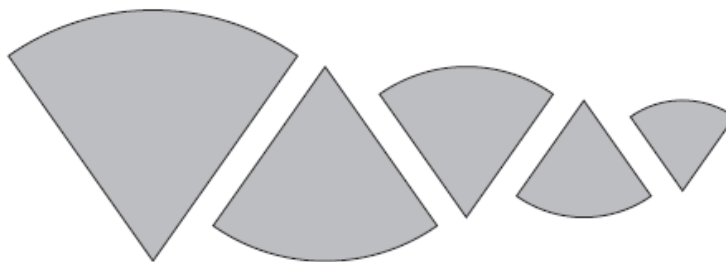


Fig. 2

- (ii) (a) Find the area of the fifth sector in the pattern. [2]
- (b) Find the total area of the first ten sectors in the pattern. [2]
- (c) Explain why the total area will never exceed a certain limit, no matter how many sectors are used, and state the value of this limit. [3]

(Total 11 marks)