Topic Y8 Functions and series (Pre-TT A) [45] MARKSCHEME

1.

Question	Scheme	Marks	AOs
11	Arithmetic sequence, $T_2 = 2k,\ T_3 = 5k - 10,\ T_4 = 7k - 14$		
	$(5k - 10) - (2k) = (7k - 14) - (5k - 10) \Rightarrow k = ...$	M1	2.1
	$\{3k - 10 = 2k - 4 \Rightarrow\}\quad k = 6$	A1	1.1b
	$\{k = 6 \Rightarrow\}\ T_2 = 12,\ T_3 = 20,\ T_4 = 28.$ So $d = 8,\ a = 4$	M1	2.2a
	$S_n = \frac{n}{2}\left(2(4) + (n-1)(8)\right)$	M1	1.1b
	$= \frac{n}{2}\left(8 + 8n - 8\right) = 4n^2 = (2n)^2$ which is a square number	A1	2.1
		(5)	
			(5 marks)

Question 11 Notes:

M1:	Complete method to find the value of *k*
A1:	Uses a correct method to find $k = 6$
M1:	Uses their value of *k* to deduce the common difference and the first term $(\neq T_2)$ of the arithmetic series.
M1:	Applies $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ with their $a \neq T_2$ and their *d*.
A1:	Correctly shows that the sum of the series is $(2n)^2$ and makes an appropriate conclusion.

2.

(i)	Attempt process for finding both critical values	**M1**	squaring both sides to obtain 3 terms on each side or considering 2 different linear eqns/inequalities
	Obtain –4	**A1**	
	Obtain $\frac{2}{3}$	**A1**	
	Attempt process for solving inequality	**M1**	table, sketch, ...; needs two critical values; implied by plausible answer≤
	Obtain □4 ≤ x ≤ $\frac{2}{3}$	**A1** **5**	With ≤ and not <
(ii)	Use correct process to find value of |x □ 2 using any value	**M1**	whether part of answer to **(i)** or not
	Obtain 2 $\frac{2}{3}$ or $\frac{8}{3}$	**A1** **2** **[7]**	**2** dependent on 5 marks awarded in part **(i)**

3.

Question	Scheme	Marks	AOs
10 (a)	$y=\dfrac{3x-5}{x+1} \Rightarrow y(x+1)=3x-5 \Rightarrow xy+y=3x-5 \Rightarrow y+5=3x-xy$	M1	1.1b
	$\Rightarrow y+5=x(3-y) \Rightarrow \dfrac{y+5}{3-y}=x$	M1	2.1
	Hence $\mathrm{f}^{-1}(x)=\dfrac{x+5}{3-x}, \quad x\in\mathbb{R},\ x\neq 3$	A1	2.5
		(3)	
(b)	$\mathrm{ff}(x)=\dfrac{3\left(\dfrac{3x-5}{x+1}\right)-5}{\left(\dfrac{3x-5}{x+1}\right)+1}$	M1	1.1a
	$=\dfrac{\dfrac{3(3x-5)-5(x+1)}{x+1}}{\dfrac{(3x-5)+(x+1)}{x+1}}$	M1	1.1b
		A1	1.1b
	$=\dfrac{9x-15-5x-5}{3x-5+x+1}=\dfrac{4x-20}{4x-4}=\dfrac{x-5}{x-1}$ (note that $a=-5$)	A1	2.1
		(4)	
(c)	$\mathrm{fg}(2)=\mathrm{f}(4-6)=\mathrm{f}(-2)=\dfrac{3(-2)-5}{-2+1}\ ;=11$	M1	1.1b
		A1	1.1b
		(2)	
(d)	$\mathrm{g}(x)=x^2-3x=(x-1.5)^2-2.25$. Hence $\mathrm{g}_{\min}=-2.25$	M1	2.1
	Either $\mathrm{g}_{\min}=-2.25$ or $\mathrm{g}(x)\geqslant -2.25$ or $\mathrm{g}(5)=25-15=10$	B1	1.1b
	$-2.25\leqslant \mathrm{g}(x)\leqslant 10$ or $-2.25\leqslant y\leqslant 10$	A1	1.1b
		(3)	
(e)	E.g. • the function g is many-one • the function g is not one-one • the inverse is one-many • $\mathrm{g}(0)=\mathrm{g}(3)=0$	B1	2.4
		(1)	
		(13 marks)	
		Question 10 Notes:	

4.

(i)		$S_{30} = {}^{30}/_{2}\,(2 \times 6 + 29 \times 1.8)$	M1	Use $d = 1.8$ in AP formula	Could be attempting S_{30} or u_{30} Formula must be recognisable, though not necessarily fully correct, so allow M1 for eg $15(6 + 29 \times 1.8)$, $15(12 + 14 \times 1.8)$ or even $15(12 + 19 \times 1.8)$ Must have $d = 1.8$ (not 1.3), $n = 30$ and $a = 6$
			A1	Correct unsimplified S_{30}	Formula must now be fully correct Allow for any unsimplified correct expression If using ${}^{1}/_{2}n(a + l)$ then l must be correct when substituted
		$= 963$	A1 [3]	Obtain 963	Units not required
(ii)		$r = {}^{7.8}/_{6} = 1.3$	M1	Use $r = 1.3$ in GP formula	Could be attempting S_N, u_N or even S_∞ Formula must be recognisable, though not necessarily fully correct Must have $r = 1.3$ (not 1.8) and $a = 6$
		$\frac{6(1-1.3^N)}{1-1.3} \le 1800$	A1	Correct unsimplified S_N	Formula must now be fully correct Allow for any unsimplified correct expression
		$1 - 1.3^N \ge -90$	M1	Link sum of GP to 1800 and attempt to rearrange to $1.3^N \le k$	Must have used correct formula for S_N of GP Allow $=$, $\ge$ or $\le$ Allow slips when rearranging, including with indices, so rearranging to $7.8^N \le k$ could get M1
		$1.3^N \le 91$ **AG**	A1	Obtain given inequality	Must have correct inequality signs throughout Correct working only, so A0 if 6×1.3^N becomes 7.8^N, even if subsequently corrected
		$N \log 1.3 \le \log 91$	M1	Introduce logs throughout and attempt to solve equation / inequality	Must be using $1.3^N \le 91$, $1.3^N = 91$ or $1.3^N \ge 91$ This M1 (and then A1) is independent of previous marks Must get as far as attempting N M0 if no evidence of use of logarithms M0 if invalid use of logarithms in attempt to solve
		$N \le 17.19$ hence $N = 17$	A1 [6]	Conclude $N = 17$	Must come from solving $1.3^N \le 91$or $1.3^N = 91$ (ie not incorrect inequality sign) Answer must be integer value Answer must be equality, so A0 for $N \le 17$ **SR** Candidates who use numerical value(s) for N can get M1 Use $r = 1.3$ in a recognisable GP formula (M0 if N is not an integer value) A1 Obtain a correct unsimplified S_N Candidates who solve $1.3^N \le 91$ and then use a value associated with their N (usually 17 and/or 18) in a GP formula will be eligible for the M1A1 for solving the inequality and also the M1A1 in the **SR** above

5.

8 (i) $\frac{1}{2} \times r^2 \times 1.2 = 60$	M1		Attempt $(\frac{1}{2})\, r^2\theta = 60$
$r = 10$	A1		Obtain $r = 10$
$r\theta = 10 \times 1.2 = 12$	B1√		State or imply arc length is $1.2r$, following their r
perimeter = 10 + 10 + 12 = 32 cm	A1	4	Obtain 32
(ii)(a) $u_5 = 60 \times 0.6^4$	M1		Attempt u_5 using ar^4, or list terms
$= 7.78$	A1	2	Obtain 7.78, or better
(b) $S_{10} = \frac{60(1-0.6^{10})}{1-0.6}$	M1		Attempt use of correct sum formula for a GP, or sum terms
$= 149$	A1	2	Obtain 149, or better (allow 149.0 – 149.2 inclusive)
(c) common ratio is less than 1, so series is convergent and hence sum to infinity exists	B1		series is convergent or $-1 < r < 1$ (allow $r < 1$) or reference to areas getting smaller / adding on less each time
$S_\infty = \frac{60}{1-0.6}$	M1		Attempt S_∞ using $\frac{a}{1-r}$
$= 150$	A1	3	Obtain $S_\infty = 150$
			SR B1 only for 150 with no method shown