

Topic Y8 Functions and series (Pre-TT B) [40]

1.

A sequence of numbers a_1, a_2, a_3, \dots is defined by

$$a_1 = 3$$
$$a_{n+1} = \frac{a_n - 3}{a_n - 2}, \quad n \in \mathbb{N}$$

(a) Find $\sum_{r=1}^{100} a_r$ (3)

(b) Hence find $\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r$ (1)

(Total 4 marks)

2.

The function f is defined by

$$f(x) = 4 + 3^{-x}, \quad x \in \mathbb{R}$$

(a) Using set notation, state the range of f [2 marks]

(b) The inverse of f is f^{-1}

(b) (i) Using set notation, state the domain of f^{-1} [1 mark]

(b) (ii) Find an expression for $f^{-1}(x)$ [3 marks]

(c) The function g is defined by $g(x) = 5 - \sqrt{x}$, ($x \in \mathbb{R} : x > 0$)

(c) (i) Find an expression for $gf(x)$ [1 mark]

(c) (ii) Solve the equation $gf(x) = 2$, giving your answer in an exact form. [3 marks]

(Total 10 marks)

3.

In a geometric series the common ratio is r and sum to n terms is S_n

Given

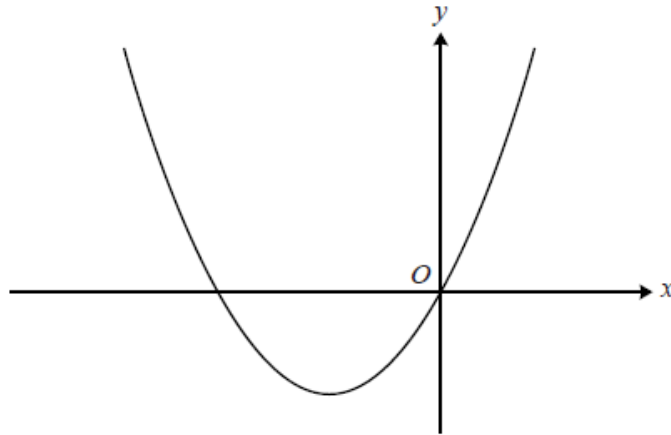
$$S_{10} = \frac{8}{7} \times S_6$$

show that $r = \pm \frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

(Total 4 marks)

4.



The function f is defined for all real values of x by

$$f(x) = k(x^2 + 4x),$$

where k is a positive constant. The diagram shows the curve with equation $y = f(x)$.

- (i) The curve $y = x^2$ can be transformed to the curve $y = f(x)$ by the following sequence of transformations:
 a translation parallel to the x -axis,
 a translation parallel to the y -axis,
 a stretch.

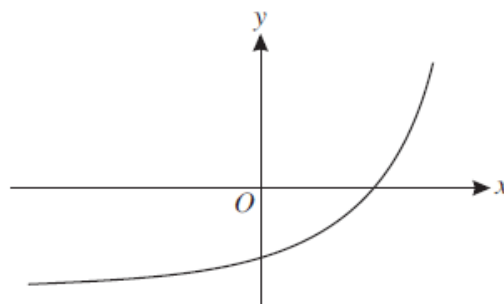
Give details, in terms of k where appropriate, of these transformations. [5]

- (ii) Find the range of f in terms of k . [2]

- (iii) It is given that there are three distinct values of x which satisfy the equation $|f(x)| = 20$. Find the value of k and determine exactly the three values of x which satisfy the equation in this case. [6]

(Total 13 marks)

5.



The diagram shows the curve $y = e^{kx} - a$, where k and a are constants.

- (i) Give details of the pair of transformations which transforms the curve $y = e^x$ to the curve $y = e^{kx} - a$. [3]

- (ii) Sketch the curve $y = |e^{kx} - a|$. [2]

- (iii) Given that the curve $y = |e^{kx} - a|$ passes through the points $(0, 13)$ and $(\ln 3, 13)$, find the values of k and a . [4]

(Total 9 marks)