

## Topic Y8 Functions and series (Pre-TT B) [40] MARKSCHEME

1.

Question	Scheme	Marks	AOs
<b>3 (a)</b>	$a_1 = 3, a_2 = 0, a_3 = 1.5, a_4 = 3$	M1	1.1b
	$\sum_{r=1}^{100} a_r = 33(4.5) + 3$	M1	2.2a
	$= 151.5$	A1	1.1b
		<b>(3)</b>	
<b>(b)</b>	$\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r = (2)(151.5) - 3 = 300$	B1ft	2.2a
		<b>(1)</b>	

2.

<b>10(a)</b>	Makes a deduction about the lower bound of the function (4)	AO2.2a	B1	The range of $f$ is the set $(x : x > 4, x \in \mathbb{R})$
	Correctly states the range of $f$ using set notation	AO2.5	B1	
<b>(b)(i)</b>	States correctly the set they gave in part (a)	AO1.2	B1F	$(x : x > 4, x \in \mathbb{R})$
<b>(b)(ii)</b>	Interchanges $x$ and $y$ at any stage	AO1.1a	M1	$y = 4 + 3^{-x}$ $x = 4 + 3^{-y}$  $3^{-y} = x - 4$ $-y = \log_3(x - 4)$  $f^{-1}(x) = -\log_3(x - 4)$
	Rearranges and takes logs	AO1.1a	M1	
	Obtains correct expression from completely correct working for $f^{-1}(x)$ , notation correct throughout	AO1.1b	A1	
<b>(c)(i)</b>	Obtains $gf(x)$	AO1.1b	B1	$gf(x) = g(4 + 3^{-x})$ $= 5 - (4 + 3^{-x})^{0.5}$

(c)(ii)	Forms equation and rearranges using 'their' $gf(x)=2$	AO1.1a	M1	$5 - (4 + 3^{-x})^{0.5} = 2$ $(4 + 3^{-x}) = 9$ $3^{-x} = 5$ $x = -\log_3 5$
	Correctly rearranges to get a single exponential term where logs can be taken. (Follow through provided 'their' equation requires the use of logs.)	AO1.1b	A1F	
	Obtains correct solution	AO1.1b	A1	
<b>Total</b>			<b>10</b>	

3.

Attempts $S_\infty = \frac{8}{7} \times S_6 \Rightarrow \frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}$	M1	2.1
$\Rightarrow 1 = \frac{8}{7} \times (1-r^6)$	M1	2.1
$\Rightarrow r^6 = \frac{1}{8} \Rightarrow r = \dots$	M1	1.1b
$\Rightarrow r = \pm \frac{1}{\sqrt[6]{8}} \quad (\text{so } k=2)$	A1	1.1b

**(4 marks)**

4.

(i)	<p>Attempt differentiation to find <math>x</math>-coordinate of stationary point or attempt completion of square as far as <math>(x + \dots)^2</math></p> <p>Obtain <math>x = -2</math> or <math>(x + 2)^2</math></p> <p>State translation by 2 in negative <math>x</math>-direction</p> <p>State translation by 4 in negative <math>y</math>-direction</p> <p>State stretch parallel to <math>y</math>-axis, scale factor <math>k</math></p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>[5]</p>	<p>or equiv; first two marks of part (i) may be earned by work seen in part (ii); <math>x = -2</math> only stated earns M1A1</p> <p>first two marks of part (i) are implied by correct answer to translation in <math>x</math>-direction or (clear) equiv; allow correct vector or (clear) equiv; allow correct vector or equiv at least mentioning <math>y</math> and <math>k</math></p>
(ii)	<p>State one of</p> <p><math>y &lt; 4k, y \leq 4k, y &lt; -4k, y \leq -4k</math></p> <p><math>y &gt; 4k, y \geq 4k, y &gt; -4k, y \geq -4k</math></p> <p>State <math>y \geq -4k</math></p>	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>allow alternative notation such as <math>f(x) \geq -4k</math> or range <math>\geq -4k</math></p>
(iii)	<p>Attempt to relate <math>y</math>-value involving <math>k</math> at their stationary point to 20 or <math>-20</math> or consider discriminant of</p> <p><math>k(x^2 + 4x) = 20</math> or of <math>k(x^2 + 4x) = -20</math></p> <p>Obtain <math>k = 5</math></p> <p>State one root <math>x = -2</math></p> <p>Attempt solution of <math>k(x^2 + 4x) = 20</math></p> <p>Obtain <math>\frac{-4 \pm \sqrt{32}}{2}</math></p> <p>Obtain <math>-2 \pm 2\sqrt{2}</math> or <math>-2 \pm \sqrt{8}</math></p>	<p>*M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1ft</p> <p>A1</p> <p>[6]</p>	<p>earned unless there is clear evidence of error in working</p> <p>dep *M; for their value of <math>k</math> provided positive or (unsimplified) exact equivs; following their value of <math>k</math></p> <p>dependent on previous A1 A1ft marks being awarded</p>

5.

(i)	<p>Refer to stretch and translation</p> <p>State stretch, factor <math>\frac{1}{k}</math>, in <math>x</math> direction</p> <p>State translation in negative <math>y</math> direction by <math>a</math></p> <p>[SC: If M0 but one transformation completely correct – B1]</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>3</p>	<p>in either order; allow here informal terms or equiv; now with correct terminology or equiv; now with correct terminology</p>
(ii)	<p>Show attempt to reflect negative part in <math>x</math>-axis</p> <p>Show correct sketch</p>	<p>M1</p> <p>A1</p> <p>2</p>	<p>ignoring curvature</p> <p>with correct curvature, no pronounced 'rounding' at <math>x</math>-axis and no obvious maximum point</p>
(iii)	<p>Attempt method with <math>x = 0</math> to find value of <math>a</math></p> <p>Obtain <math>a = 14</math></p> <p>Attempt to solve for <math>k</math></p> <p>Obtain <math>k = 3</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>4</p> <p>9</p>	<p>... other than (or in addition to) value <math>-12</math> and nothing else</p> <p>using any numerical <math>a</math> with sound process</p>