Write yours and your teacher's name at the top of your answer sheets.

U6 Mathematics Mock

Paper 1 (Teacher X)
February 2019
2018-2019

Duration: 1 hour 30 minutes

Total number of marks: 74

Write your answers in the spaces provided.
Additional paper may be used if necessary.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

1.

The equation $x^3 - 3x + 1 = 0$ has three real roots.

(a) Show that one of the roots lies between –2 and –1

[2 marks]

(b) Taking $x_1 = -2$ as the first approximation to one of the roots, use the Newton-Raphson method to find x_2, the second approximation.

[3 marks]

(c) Explain why the Newton-Raphson method fails in the case when the first approximation is $x_1 = -1$

[1 mark]

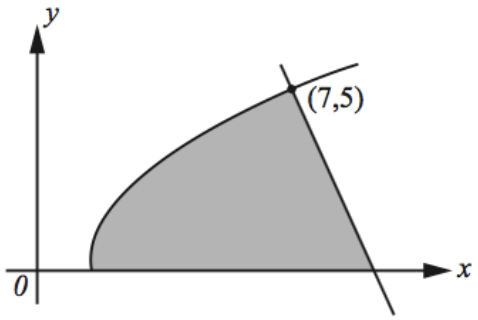
2.

Use integration to find the exact value of $\int_{\frac{1}{16}\pi}^{\frac{1}{8}\pi} (9 - 6\cos^2 4x)\,dx$. **[5]**

3.

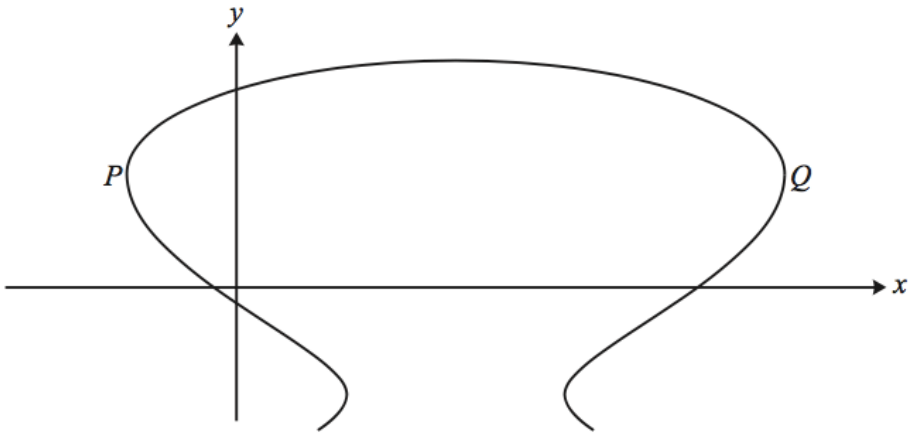
The equation of a curve is $y = e^{2x}\cos x$. Find $\frac{dy}{dx}$ and hence find the coordinates of any stationary points for which $-\pi \leqslant x \leqslant \pi$. Give your answers correct to 3 significant figures. **[6]**

4.



The diagram shows the curve $y = \sqrt{4x-3}$ and the normal to the curve at the point $(7,5)$. The shaded region is bounded by the curve, the normal and the x-axis. Find the exact area of the shaded region. **[8]**

5.



The diagram shows the curve with equation $x^2 + y^3 - 8x - 12y = 4$. At each of the points P and Q the tangent to the curve is parallel to the y-axis. Find the coordinates of P and Q. **[8]**

6.

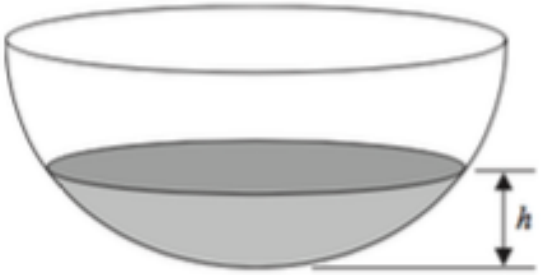


Figure 3

A bowl is modelled as a hemispherical shell as shown in Figure 3.

Initially the bowl is empty and water begins to flow into the bowl.

When the depth of the water is h cm, the volume of water, V cm^3, according to the model is given by

$$V = \frac{1}{3}\pi h^2(75 - h), \qquad 0 \leqslant h \leqslant 24$$

The flow of water into the bowl is at a constant rate of 160π cm^3 s^{-1} for $0 \leqslant h \leqslant 12$

(a) Find the rate of change of the depth of the water, in cm s^{-1}, when $h = 10$

(5)

Given that the flow of water into the bowl is increased to a constant rate of 300π cm^3 s^{-1} for $12 < h \leqslant 24$

(b) find the rate of change of the depth of the water, in cm s^{-1}, when $h = 20$

(2)

7.

A container in the shape of an inverted cone of radius 3 metres and vertical height 4.5 metres is initially filled with liquid fertiliser. This fertiliser is released through a hole in the bottom of the container at a rate of $0.01\,\text{m}^3$ per second. At time t seconds the fertiliser remaining in the container forms an inverted cone of height h metres.

[The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.]

(i) Show that $h^2 \frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{9}{400\pi}$. **[5]**

(ii) Express h in terms of t. **[4]**

(iii) Find the time it takes to empty the container, giving your answer to the nearest minute. **[2]**

8.

A curve is defined by the parametric equations $x = \frac{2t}{1+t}$ and $y = \frac{t^2}{1+t}$, $t \neq -1$.

(i) **(a)** Show that the curve passes through the origin. **[1]**

(b) Find the y-coordinate when $x = 1$. **[1]**

(ii) Show that the area enclosed by the curve, the x-axis and the line $x = 1$ is given by

$$\int_0^1 \frac{2t^2}{(1+t)^3}\,\mathrm{d}t.$$

[5]

(iii) **In this question you must show detailed reasoning.**

Hence use an appropriate substitution to find the exact area enclosed by the curve, the x-axis and the line $x = 1$. **[6]**

Mechanics

9.

A football is kicked from horizontal ground with speed $20\,\text{m s}^{-1}$ at an angle of $\theta°$ above the horizontal. The greatest height the football reaches above ground level is 2.44 m. By modelling the football as a particle and ignoring air resistance, find

(i) the value of θ, **[2]**

(ii) the range of the football. **[2]**

10.

At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}

Find the speed of P when $t = 4$

(6)