U6 Mock Teacher X 18-19 SOLUTIONS [74]

1.

(a)	Starts an argument by showing that $f(-2) < 0$ and $f(-1) > 0$ Both attempted and at least one evaluated correctly f must be clearly defined or substitution of values must be explicit.	AO2.1	R1	$f(x) = x^3 - 3x + 1$ $f(-2) = (-2)^3 - 3(-2) + 1 = -1 < 0$ $f(-1) = (-1)^3 - 3(-1) + 1 = 3 > 0$ Change of sign and $f(x)$ is continuous so a root must lie between $x = -2$ and $x = -1$
	Explains reasoning fully to complete the argument Evaluations above need to be of opposite sign **and** 'change of sign' OE seen **and** reference to x-values –2 & –1 **and** reference to continuous function	AO2.4	E1	
(b)	Uses Newton–Raphson, must have $f'(x)$ correct PI by correct substitution	AO1.1a	M1	$x_{n+1} = x_n - \frac{(x_n)^3 - 3x_n + 1}{3(x_n)^2 - 3}$
	Substitutes $x_1 = -2$ into 'their' Newton–Raphson formula (accept 'their' $f(-2)$ from part **(a)**)	AO1.1a	M1	$x_2 = -2 - \frac{-1}{3(-2)^2 - 3}$
	Obtains correct value for x_2 $-\frac{17}{9}$ or $-1\frac{8}{9}$ or -1.89 or better	AO1.1b	A1	$= -\frac{17}{9}$
(c)	Explains why the method fails when $x_1 = -1$ This must include a substitution of $x_1 = -1$ and an explanation of what goes wrong eg division by zero | not possible gradient zero | method fails	AO2.4	E1	$x_1 = -1$ $x_2 = -2 = \frac{3}{3(-1)^2 - 3} = -2 - \frac{3}{0}$ causes division by zero (expression undefined) **ALT** $f'(-1) = 0$, function has zero gradient at this point, method will fail
	Total		**6**	

[6 marks]

2.

2			$\cos 8x$ seen in integrand	**M1**		
			$F[x] = Ax + B\sin 8x$ oe	**M1***	*A* and *B* are non-zero constants	
			$F[x] = 6x - \frac{3}{8}\sin 8x$	**A1**		
			$F[\frac{1}{8}\pi] - F[\frac{1}{16}\pi]$	**M1*dep**		
			$\frac{3}{8}\pi + \frac{3}{8}$ oe	**A1**		allow eg $0.375\pi + 0.375$ or fractions not in lowest terms
				[5]		

[5 marks]

3.

3			$\frac{dy}{dx} = \pm k e^{2x}\cos x \pm e^{2x}\sin x$	**M1***	*k* is any constant	Product Rule
			$\frac{dy}{dx} = 2e^{2x}\cos x - e^{2x}\sin x$ oe	**A1**		
			their $\frac{dy}{dx} = 0$	**M1dep***		
			$\tan x = 2$ or $\cos x = (\pm)\frac{1}{\sqrt{5}}$ or $\sin x = (\pm)\frac{2}{\sqrt{5}}$	**A1**	ignore omission of "$e^{2x} = 0$ has no solution",	or $\sqrt{5}\cos(x + \tan^{-1}\frac{1}{2}) = 0$
			$x = 1.11$ and -2.03 cao	**A1**	(1.11, 4.09) and / or (– 2.03 , – 0.00765)	if **A0A0**, **SC1** for all 4 values to greater precision 1.107…, – 2.034…, 4.094…., – 0.0076457…(or – 0.007646)
			$y = 4.09$ and -0.00765 cao	**A1**	or **A1** for each correct pair of co-ordinates: mark to benefit of candidate	NB $x = 1.107148718$ and -2.034443936 $y = 4.094229238$ and -0.007645738
					extra values within range incur a penalty of one mark; **or** any finite value for *x* obtained from $e^{2x} = 0$ incurs a penalty of one mark	ignore extra values outside range
				[6]		

[6 marks]

4.

Differentiate to obtain $k(4x-3)^{-\frac{1}{2}}$	M1	For any non-zero constant *k*
Obtain correct $2(4x-3)^{-\frac{1}{2}}$	A1	Or unsimplified equiv
Use negative reciprocal of gradient to find intersection of normal with *x*-axis	M1	Using their attempt at first derivative; <u>either</u> using equation of normal ($y = -\frac{5}{2}x + \frac{45}{2}$) <u>or</u> relevant right-angled triangle
Obtain $-\frac{5}{2}$ for gradient of normal and hence $x = 9$ or equiv such as base of triangle is 2	A1	
Integrate to obtain $p(4x-3)^{\frac{3}{2}}$	M1	For any non-zero constant *p*
Obtain correct $\frac{1}{6}(4x-3)^{\frac{3}{2}}$	A1	Or unsimplified equiv
Use limits $\frac{3}{4}$ and 7 to obtain $\frac{125}{6}$ for area under curve	A1	Allow calculation apparently using only upper limit
Use triangle area to obtain $\frac{155}{6}$ for shaded area	A1 **[8]**	

[8 marks]

5.

6	$3y^2\frac{dy}{dx}$	B1	or $2x\frac{dx}{dy}$	if B0B0 M0
	$2x-12\frac{dy}{dx}-8$	B1	$3y^2-8\frac{dx}{dy}-12$	SC2 for $\frac{dy}{dx}=$ $\frac{1}{3}(-x^2+8x+12y+4)^{\frac{-2}{3}}\times(-2x+8+12\frac{dy}{dx})$
	their $3y^2\frac{dy}{dx}-12\frac{dy}{dx}=8-2x$ soi must be two terms on each side and must follow from RHS $=0$	M1	their $2x\frac{dx}{dy}-8\frac{dx}{dy}=-3y^2+12$ must be two terms on each side must follow from RHS $=0$	M1 may be earned for setting correct denominator equal to 0
	$\frac{dy}{dx}=\frac{8-2x}{3y^2-12}$ oe	A1	This mark may be implied if $\frac{dx}{dy}=0$ is substituted and there is no evidence for an incorrect expression for $\frac{dx}{dy}$	
	their $3y^2-12=0$	M1*		$x\neq 4$ not required
	$y=(\pm)\,2$	A1	A0 if $\frac{dy}{dx}$ incorrect	
	substitution of their positive y value in original equation	M1dep*		ignore substitution of -2
	$x=10, x=-2$ and no others cao	A1 [8]	A0 if $\frac{dy}{dx}$ incorrect	condone omission of formal statement of coordinates (10, 2) and (–2, 2)

[8 marks]

6.

8(a)	$\frac{dV}{dt}=160\pi,\ V=\frac{1}{3}\pi h^2(75-h)=25\pi h^2-\frac{1}{3}\pi h^3$		
	$\frac{dV}{dh}=50\pi h-\pi h^2$	M1 A1	1.1b 1.1b
	$\left\{\frac{dV}{dh}\times\frac{dh}{dt}=\frac{dV}{dt}\Rightarrow\right\}\ (50\pi h-\pi h^2)\frac{dh}{dt}=160\pi$	M1	3.1a
	When $h=10$, $\left\{\frac{dh}{dt}=\frac{dV}{dt}\div\frac{dV}{dh}\Rightarrow\right\}\ \frac{160\pi}{50\pi(10)-\pi(10)^2}\ \left\{=\frac{160\pi}{400\pi}\right\}$	dM1	3.4
	$\frac{dh}{dt}=0.4\ (\text{cm s}^{-1})$	A1	1.1b
		(5)	
(b)	$\frac{dh}{dt}=\frac{300\pi}{50\pi(20)-\pi(20)^2}$	M1	3.4
	$\frac{dh}{dt}=0.5\ (\text{cm s}^{-1})$	A1	1.1b
		(2)	
			(7 marks)

[7 marks]

7.

10	(i)		$\frac{dV}{dt} = \pm 0.01$	B1		
			by similar triangles, $\frac{h}{4.5} = \frac{r}{3}$	B1	may be implied by $r = \frac{2h}{3}$ oe	
			$\frac{dV}{dh} = \frac{4}{9}\pi h^2$ oe	B1		
			$\frac{dh}{dt} = \pm 0.01 \times$ their $\frac{dh}{dV}$ oe	M1	use of Chain rule	may follow from incorrect differentiation: expressions must be a function of either r or h or both
			$-0.01 = (\frac{4}{9}\pi h^2) \times \frac{dh}{dt}$	A1 [5]	completion to given result www	$h^2 \frac{dh}{dt} = \frac{-0.09}{4\pi} = \frac{-9}{400\pi}$
10	(ii)		$\int h^2 dh = \int \frac{-9}{400\pi} dt$ oe soi	M1	separation of variables	if no subsequent work, integral signs needed, but allow omission of dh or dt, but must be correctly placed if present;
			$\frac{h^3}{3} = \frac{-9}{400\pi} t (+c)$	A1		
			substitution of $t = 0$ and $h = 4.5$ in their expression following integration	M1	expression must include c and powers must be correct on each side	
			$h = \sqrt[3]{\frac{729}{8} - \frac{27t}{400\pi}}$ oe isw	A1 [4]	allow – 0.0215 or – 0.02148591…r.o.t to 4 sf or more and similarly 91.125	$91.125 = {}^{729}/_8$
10	(iii)		set $h = 0$ and solve to obtain positive t	**M1**	or $(t =) \frac{1}{3}\pi \times 3^2 \times 4.5 \div 0.01$ $(= 1350\pi)$	NB $1350\pi = 4241.150082\ldots$
			71 minutes cao	**A1** **[2]**		

[11 marks]

8.

(i)	(a)	when $x = 0$, $t = 0$ and hence $y = 0$	**E1** **[1]**	**2.4**	Justify (0, 0) convincingly	
	(b)	when $x = 1$, $t = 1$ and hence $y = 0.5$	**B1** **[1]**	**1.1**	Obtain $y = 0.5$	
(ii)		$\frac{dx}{dt} = \frac{2}{(1+t)^2}$	**M1**	**2.1**	Attempt $\frac{dx}{dt}$	Using quotient rule, or other valid method
			A1	**2.1**	Obtain correct derivative	
		$\int \frac{t^2}{1+t} dx = \int \frac{t^2}{1+t} \times \frac{2}{(1+t)^2} dt$	**M1**	**2.1**	Use $\int y dx = \int y \frac{dx}{dt} dt$	
		$= \int \frac{2t^2}{(1+t)^3} dt$	**A1**	**2.1**	Obtain given answer	
			B1 **[5]**	**2.4**	Justify t-limits from $x = 0, 1$	$x = 0$: $\frac{2t}{1+t} = 0$ so $t = 0$ $x = 1$: $\frac{2t}{1+t} = 1$ $2t = 1 + t$ so $t = 1$
(iii)		**DR** use $u = 1 + t$ giving $du = dt$	**E1**	**1.1a**	Must be stated explicitly	
		$\int \frac{2t^2}{(1+t)^3} dt = \int \frac{2(u-1)^2}{u^3} du$	**M1**	**1.1a**	Attempt to change integrand to function of u	
		$= \int 2u^{-1} - 4u^{-2} + 2u^{-3} du$	**A1**	**1.1**	Obtain correct integrand	Any equivalent form
		$= \left[2\ln u + 4u^{-1} - u^{-2}\right]_1^2$	**M1**	**1.1a**	Attempt integration	
		$= (2\ln 2 + 2 - 0.25) - (2\ln 1 + 4 - 1)$	**M1**	**1.1a**	Attempt use of limits $u = 1, 2$	
		$= 2\ln 2 - \frac{5}{4}$	**A1** **[6]**	**1.1**	Obtain correct exact area	Allow any exact equiv

[13 marks]

9.

1	(i)		$(20\sin\theta)^2 - 2g(2.44) = 0$ $\theta = 20.2$	M1 A1 **[2]**	Use $v^2 = u^2 + 2as$ vertically with $v = 0$ $\theta = 20.22908\ldots$
	(ii)		$20\sin \text{cv}(\theta)t - 1/2gt^2 = 0$ AND range = $20\,\text{cv}(t)\cos \text{cv}(\theta)$ Range = 26.5 m	M1 A1 **[2]**	Use $s = ut + ½at^2$ vertically with $s = 0$ OR use $v = u + at$ and doubles t AND horizontally with time found from vertical. (t = 1.4113… s or 1.4093…s (from 20.2)) Range = 26.48541… m or 26.45387….m (from 20.2)
		OR	$\dfrac{20^2\sin(2\times \text{cv}(\theta))}{g}$ Range = 26.5 m	M1 A1 **[2]**	Use of range formula Range = 26.48541… m or 26.45387….m (from 20.2)

[4 marks]

10.

Integrate **a** w.r.t. time	M1	1.1a
$\mathbf{v} = \dfrac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of **C**)	A1	1.1b
$\mathbf{v} = \dfrac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\,\mathbf{j}$	M1	1.1b
Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
Speed = 100 m s^{-1}	A1ft	1.1b
		(6 marks)

Notes:

1st **M1:** for integrating **a** w.r.t. time (powers of t increasing by 1)
1st **A1:** for a correct **v** expression without **C**
2nd **A1:** for a correct **v** expression including **C**
2nd **M1:** for putting $t = 4$ into their **v** expression
3rd **M1:** for finding magnitude of their **v**
3rd **A1:** **ft** for 100 m s^{-1}, follow through on an incorrect **v**

[6 marks]