

Write yours and your teacher's name at the top of your answer sheets.

U6 Mathematics Mock
Paper 1 (X – CFS/JUD/JLH/NDW)
February 2020
2019-2020

Duration: 1 hour 30 minutes

Total number of marks: 69

*Write your answers in the spaces provided.
Additional paper may be used if necessary.*

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

1.

The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n} \quad (3)$$

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3 (2)

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

(1)

[6 marks]

2.

A curve has equation $(x+y)^2 = xy^2$. Find the gradient of the curve at the point where $x = 1$. [7]

[7 marks]

3.

(a) Given that $\log_a y = 2 \log_a 7 + \log_a 4 + \frac{1}{2}$, find y in terms of a .

[4 marks]

(b) When asked to solve the equation

$$2 \log_a x = \log_a 9 - \log_a 4$$

a student gives the following solution:

$$2 \log_a x = \log_a 9 - \log_a 4$$

$$\Rightarrow 2 \log_a x = \log_a \frac{9}{4}$$

$$\Rightarrow \log_a x^2 = \log_a \frac{9}{4}$$

$$\Rightarrow x^2 = \frac{9}{4}$$

$$\therefore x = \frac{3}{2} \text{ or } -\frac{3}{2}$$

Explain what is wrong with the student's solution.

[1 mark]

[5 marks]

4.

The volume of a spherical bubble is increasing at a constant rate.

Show that the rate of increase of the radius, r , of the bubble is inversely proportional to r^2

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

[4 marks]

[4 marks]

5.

A curve has parametric equations

$$x = 2 \sin t, \quad y = \cos 2t + 2 \sin t$$

for $-\frac{1}{2}\pi \leq t \leq \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = 1 - 2 \sin t$ and hence find the coordinates of the stationary point. **[5]**

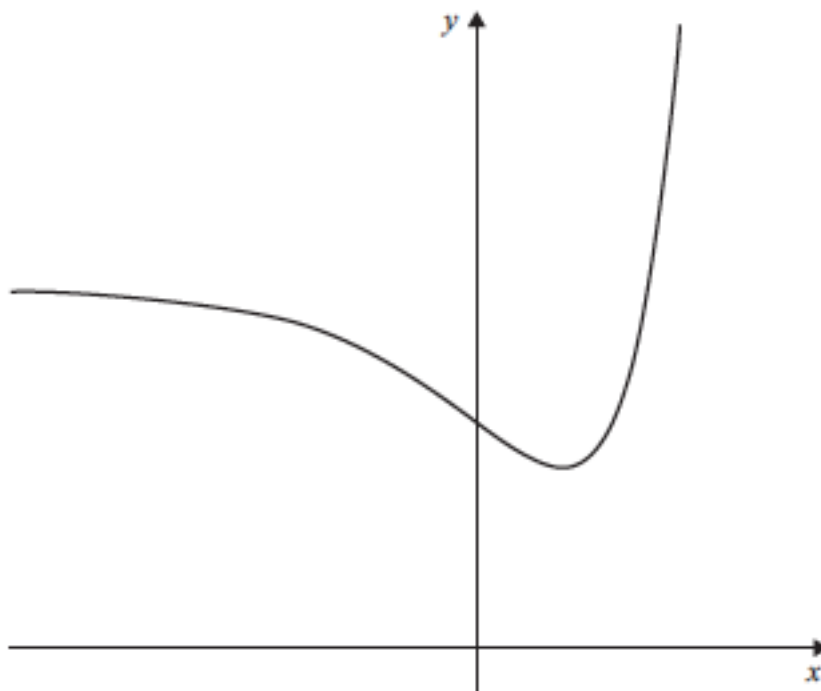
(ii) Find the cartesian equation of the curve. **[3]**

[8 marks]

6.

A function f has domain \mathbb{R} and range $\{y \in \mathbb{R} : y \geq e\}$

The graph of $y = f(x)$ is shown.



The gradient of the curve at the point (x, y) is given by $\frac{dy}{dx} = (x - 1)e^x$

Find an expression for $f(x)$.

Fully justify your answer.

[8 marks]

[8 marks]

7.

Show that

$$\int_0^2 2x\sqrt{x+2} \, dx = \frac{32}{15}(2 + \sqrt{2})$$

(7)

[7 marks]

8.

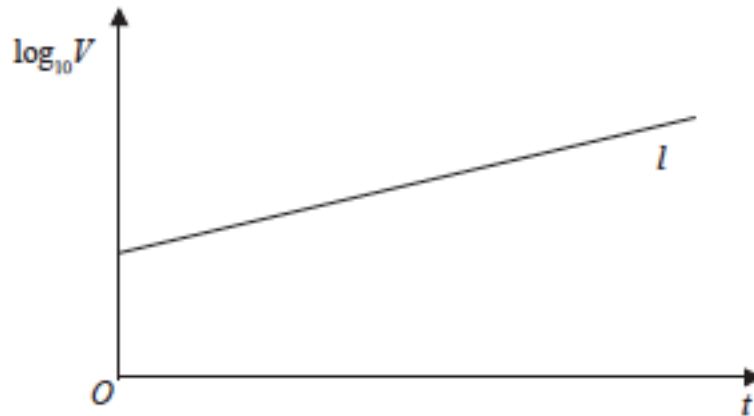


Figure 3

The value of a rare painting, £ V , is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10} V$ since 1st January 1980.

The equation of line l is $\log_{10} V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q .

(4)

(b) With reference to the model interpret

(i) the value of the constant p ,

(ii) the value of the constant q .

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

[8 marks]

9.

The height above ground, H metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{dH}{dt} = \frac{H \cos(0.25t)}{40}$$

where t is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

(a) show that $H = 5e^{0.1 \sin(0.25t)}$ (5)

(b) State the maximum height of the passenger above the ground. (1)

The passenger reaches the maximum height, for the second time, T seconds after the start of the ride.

(c) Find the value of T . (2)

[8 marks]

Mechanics

10.

[In this question \mathbf{i} and \mathbf{j} are horizontal unit vectors due east and due north respectively and position vectors are given relative to the fixed point O .]

A particle P moves with constant acceleration.

At time $t = 0$, the particle is at O and is moving with velocity $(2\mathbf{i} - 3\mathbf{j})\text{ m s}^{-1}$

At time $t = 2$ seconds, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})\text{ m}$.

(a) Show that the magnitude of the acceleration of P is 2.5 m s^{-2} (4)

At the instant when P leaves the point A , the acceleration of P changes so that P now moves with constant acceleration $(4\mathbf{i} + 8.8\mathbf{j})\text{ m s}^{-2}$

At the instant when P reaches the point B , the direction of motion of P is north east.

(b) Find the time it takes for P to travel from A to B . (4)

[8 marks]