

Write yours and your teacher's name at the top of your answer sheets.

U6 Mathematics Mock

Paper 2 (Teacher Y)

February 2019

2018-2019

Duration: 1 hour 30 minutes

Total number of marks: 73

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

1.

A geometric progression has first term 3 and second term -6 .

(i) State the value of the common ratio. [1]

(ii) Find the value of the eleventh term. [2]

(iii) Find the sum of the first twenty terms. [2]

2.

By first using appropriate identities, solve the equation

$$5 \cos 2\theta \operatorname{cosec} \theta = 2$$

for $0^\circ < \theta < 180^\circ$. [6]

3.

When θ is small, find an approximation for $\cos 3\theta + \theta \sin 2\theta$, giving your answer in the form $a + b\theta^2$

[3 marks]

4.

(i) Sketch the graph of $y = |2x - 7a|$, where a is a positive constant. State the coordinates of the points where the graph meets each axis. [2]

(ii) Solve the inequality $|2x - 7a| < 4a$. [3]

5.

The functions f and g are defined for all real values of x by

$$f(x) = 2x^3 + 4 \quad \text{and} \quad g(x) = \sqrt[3]{x - 10}.$$

(i) Evaluate $f^{-1}(-50)$. [2]

(ii) Show that $fg(x) = 2x - 16$. [2]

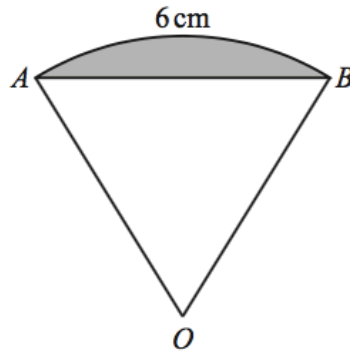
(iii) Differentiate $gf(x)$ with respect to x . [3]

6.

(i) Find the first three terms in the binomial expansion of $(8 - 9x)^{\frac{2}{3}}$ in ascending powers of x . [4]

(ii) State the set of values of x for which this expansion is valid. [1]

7.



The diagram shows a sector AOB of a circle with centre O . The length of the arc AB is 6 cm and the area of the sector AOB is 24 cm^2 . Find the area of the shaded segment enclosed by the arc AB and the chord AB , giving your answer correct to 3 significant figures. [6]

8.

Express $\frac{2+x^2}{(1+2x)(1-x)^2}$ in partial fractions and hence show that $\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2} dx = \frac{1}{2} \ln \frac{3}{2} + \frac{1}{3}$. [9]

9.

Angela makes the following claim.

“ n is an odd positive integer greater than 1 $\Rightarrow 2^n - 1$ is prime”

Prove that Angela’s claim is false. [4]

10.

An arithmetic progression u_1, u_2, u_3, \dots is defined by $u_1 = 5$ and $u_{n+1} = u_n + 1.5$ for $n \geq 1$.

(i) Given that $u_k = 140$, find the value of k . [3]

A geometric progression w_1, w_2, w_3, \dots is defined by $w_n = 120 \times (0.9)^{n-1}$ for $n \geq 1$.

(ii) Find the sum of the first 16 terms of this geometric progression, giving your answer correct to 3 significant figures. [2]

11.

- (a) Express $10 \cos \theta - 3 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$.
Give the exact value of R and give the value of α , in degrees, to 2 decimal places.

(3)

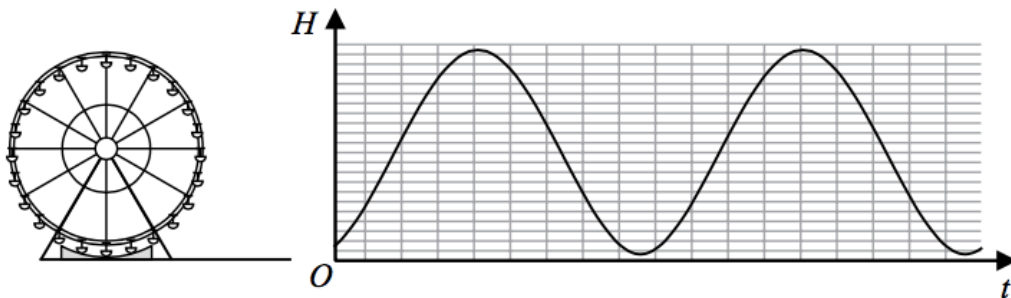


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10 \cos(80t)^\circ + 3 \sin(80t)^\circ$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

- (b) (i) find a complete equation for the model,
(ii) hence find the maximum height of the passenger above the ground.

(2)

- (c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

- (d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

Statistics

12.

- (i) Events A and B are independent, and $P(A \cap B) = \frac{1}{24}$ and $P(A \cup B) = \frac{3}{8}$.

Find $P(A)$ and $P(B)$.

[5]

- (ii) Events C and D are such that $P(C) = 0.6$, $P(D) = 0.3$ and $P(C \cup D) = 0.8$. Find $P(D|C')$.

[4]