U6 Mock Teacher Y 18-19 SOLUTIONS [73]

1.

1	(i)		$r = -2$	B1 [1]	State –2	Not $^{-6}/_3$ as final answer No need to see $r = \ldots$, and also condone other variables
	(ii)		$3 \times (-2)^{10} = 3072$	M1 A1 [2]	Attempt u_{11} Obtain 3072	Must be using correct formula, with $a = 3$ and $r = -2$, or their r from (i) Allow M1 for 3×-2^{10} Using $r = 2$ is M0, unless this was their value in (i) Allow M1 for listing terms as far as u_{11} CWO Allow A1 BOD for $3 \times -2^{10} = 3072$ If listing terms, then need to indicate that 3072 is the required value
	(iii)		$\frac{3(1-(-2)^{20})}{1-(-2)} = -1048575$	M1 A1 [2]	Attempt S_{20} Obtain –1048575	Must be using correct formula, with $a = 3$ and $r = -2$, or their r from (i) Allow M1 for correct formula, but with no brackets around the -2 Allow M1 for attempting to sum first 20 terms Allow M1 for $\frac{3(1+2^{20})}{1+2}$ as long as correct general formula is also seen Could also come from manually summing terms **NB** $\frac{3(1--2^{20})}{1--2}$ gives 1048577

[5 marks]

2.

2			State or imply $\operatorname{cosec} q = 1$, $\sin q$ Attempt to express equation in terms of $\sin q$ only Obtain $10\sin^2 q + 2\sin q - 5 = 0$ Attempt use of formula to find $\sin q$ from 3-term quadratic equation involving $\sin q$ (using formula or completing square even if their equation can be solved by factorisation) Obtain 37.9° Obtain 142°	B1 M1 A1 M1 A1 A1 [6]	allow $\operatorname{cosec} = 1$, sin using identity of form $-1 - 2\sin^2 q$ for $\cos 2q$ or unsimplified equiv involving $\sin q$ only but with no $\sin q$ remaining in denominator use implied by at least one correct value of $\sin q$ or q; if correct quadratic formula quoted, condone one sign error for M1; if formula not first quoted, any error leads to M0 or greater accuracy 37.8896… or greater accuracy 142.1103…; and no others between 0 and 180; ignore any answers, right or wrong, outside 0 - 180	 if completion of square used to solve equation, this must be correct for M1 to be earned no working and answers only (max 2/6): 37.9 (or greater accuracy) B1 142 (or greater accuracy) and no others … B1

[6 marks]

3.

Uses either $\cos x \approx 1 - \frac{1}{2}x^2$ or $\sin x \approx x$ (PI)	AO1.2	B1	$\cos 3\theta + \theta \sin 2\theta \approx 1 - \frac{(3\theta)^2}{2} + \theta(2\theta)$ $\approx 1 - \frac{5}{2}\theta^2$
Substitutes 2θ and 3θ into 'their' expression	AO1.1a	M1	
Obtains correct answer	AO1.1b	A1	
Total		**3**	

[3 marks]

4. STUDENTS DID NOT DO PART (iii)

i		Draw V-shaped graph with vertex on positive x-axis	B1	And graph extending at least a little into second quadrant; condone minimal smoothing at the vertex; allow graph which is asymmetrical about vertical line through vertex unless it is an extreme case
		State $(\frac{7}{2}a, 0)$ and $(0, 7a)$	B1 [2]	Can be earned if first B1 not awarded; allow for $\frac{7}{2}a$ and $7a$ marked on axes of graph or cases where zero coordinates are not given but are clearly implied
ii		Attempt to find two critical values	M1	By squaring both sides (giving 3 terms on left) and solving quadratic equation <u>or</u> by solving two linear equations (one with signs of $2x$ and $4a$ the same and one with the signs different) <u>or</u> using graph with horizontal line representing $y = 4a$
		Obtain $\frac{3}{2}a$ and $\frac{11}{2}a$	A1	
		Conclude with $\frac{3}{2}a < x < \frac{11}{2}a$	A1 [3]	Allow the logically correct '$x > \frac{3}{2}a$ and $x < \frac{11}{2}a$' but not conclusions such as '$x > \frac{3}{2}a,\ x < \frac{11}{2}a$'; giving a a particular value means only M1 is available; use of $\le$ signs is final A0
iii		Relate $\ln N$ to their upper limit of **(ii)** with $a = 1.5$ <u>or</u> proceed directly from inequality in **(iii)** to $2\ln N < 16.5$	M1	
		State the single value 3827	A1 [2]	A0 for $N \le 3827$; A0 for $N < 3827.6$

[5 marks]

5.

4	(i)		<u>Either</u>: State $2x^3 + 4 = -50$ State -3 and no other	B1 B1		
			<u>Or</u>: Obtain $\sqrt[3]{\frac{1}{2}(x-4)}$ for inverse of f State -3 and no other	B1 B1 [2]	or equiv; using any letter	
4	(ii)		Show composition of functions the right way round Obtain $2x - 16$	M1 A1 [2]	AG; necessary detail needed	first step $2(x-10)+4$ acceptable but then two more steps needed
4	(iii)		Obtain $\sqrt[3]{2x^3 - 6}$ or $(2x^3 - 6)^{\frac{1}{3}}$ for $\text{gf}(x)$	B1	or unsimplified equiv	
			Apply chain rule to function which is cube root of a non-linear expression	M1	condone incorrect constant; otherwise use of chain rule for their function must be correct	may use $u = 2x^3 - 6$; M1 earned for expression involving u
			Obtain $2x^2(2x^3 - 6)^{-\frac{2}{3}}$	A1 [3]	or similarly simplified equiv; do not accept final answer with $\frac{6}{3}$ unsimplified	… in terms of x

[7 marks]

6.

4	(i)		$8^{2/3} = 4$	**B1**		may be embedded
			$(1 - \frac{9x}{8})^{2/3}$ seen	**M1**	$8^{2/3} + (2/3)8^{-1/3}(\pm 9x) + \frac{2/3 \times (2/3 - 1)}{2!} 8^{-4/3}(\pm 9x)^2$	ignore extra terms
			$1 + \left(\frac{2}{3}\right)\left(\frac{\pm 9x}{k}\right) + \frac{1}{2!}\left(\frac{2}{3}\right)\left(\frac{2}{3} - 1\right)\left(\frac{\pm 9x}{k}\right)^2$ where k is an integer greater than 1	**M1**	$4 + (2/3)(\frac{1}{2})(\pm 9x) + \frac{2/3 \times (2/3 - 1)}{2!}(\frac{1}{16})(\pm 9x)^2$	or better
			$4 - 3x - \frac{9}{16}x^2$ or $4(1 - \frac{3}{4}x - \frac{9}{64}x^2)$ cao	**A1** **[4]**		
4	(ii)		$-\frac{8}{9} < x < \frac{8}{9}$ or $\|x\| < 8/9$ isw cao	**B1** **[1]**		

[5 marks]

7.

$r\theta = 6$ $½r^2\theta = 24$ $½r \times 6 = 24$ $r = 8, \theta = 0.75$ segment area = $24 - ½ \times 8^2 \times \sin 0.75$ $= 2.19$	B1* B1*	State $r\theta = 6$ State $½r^2\theta = 24$	Or exact equiv from using a fraction of the circle Or exact equiv from using a fraction of the circle Allow B1 for $^1/_2\, r \times$ arc = 24 Stating both $½r^2\theta = 24$ and $^1/_2r^2\sin\theta = 24$ is B0 unless only the correct equation is subsequently used B1 B1 can be implied by a correct equation in a single variable
	M1d*	Attempt to solve simultaneously to find r or θ	As far as attempting r or θ, using a valid method (but allow slips) Must be using the two correct equations in r and θ
	A1	Obtain $r = 8, \theta = 0.75$ (aef)	Both values required
	M1	Attempt area of segment	24 – area of triangle, using $^1/_2r^2\sin\theta$ or equiv Allow if evaluated in degree mode (gives 23.58) Allow M1 for attempting $^1/_2r^2(\theta - \sin\theta)$ with their r and θ, even if this does not give area of sector as 24
	A1	Obtain 2.19, or better	Allow final answer in range [2.187, 2.188] if > 3sf
	[6]		Could use variables other than r and θ **Alt method for working in degrees** B1 - state $^\theta/_{360} \times 2\pi r = 6$ B1 - state $^\theta/_{360} \times \pi r^2 = 24$ M1 - attempt to solve simultaneously A1 - obtain $r = 8$, $\theta = 43.0^\circ$ or better (42.97...) M1 - attempt area of segment NB using $^1/_2r^2(\theta - \sin\theta)$ with θ in degrees is M0 as incorrect attempt at area of sector A1 - obtain 2.19 or better

[6 marks]

8.

9			$\frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$ may be seen in later work	B1	or $\frac{A}{1+2x} + \frac{Bx+C}{(1-x)^2}$ may be seen later in later work	if B0M0, SC1 for $\frac{1}{1+2x}$ seen
			$2 + x^2 \equiv A(1-x)^2 + B(1+2x)(1-x) + C(1+2x)$ $A = 1, B = 0$ and $C = 1$ www	M1 A1A1A1	or $A(1-x)^2 + (Bx + C)(1 + 2x)$	allow only sign errors , not algebraic errors
			$\int \left(\frac{1}{1+2x} + \frac{1}{(1-x)^2} \right) dx =$ $a\ln(1 + 2x) + b(1-x)^{-1}$	M1*	a and b are non-zero constants	ignore extra terms
			$F(x) = ½\ln(1 + 2x) + (1-x)^{-1}$	A1		
			their $\frac{1}{2}\ln(\frac{3}{2}) + \frac{4}{3} - (\frac{1}{2}\ln 1 + 1)$	M1dep*		
			$\frac{1}{2}\ln(\frac{3}{2}) + \frac{4}{3} - 0 - 1$	A1 [9]	and completion to given result www	NB $\frac{1}{2}\ln(\frac{3}{2}) + \frac{1}{3}$

[9 marks]

9.

Attempt $2^n - 1$ for any odd integer n eg $2^9 - 1 = 511$ This is a counter example as 511 is divisible by 7, hence claim false	**M1** **A1** **M1** **E1** **[3]**	**3.1a** **2.1** **1.1** **2.2a**	 Any $2^{\text{odd}} - 1$ that is non-prime Counter example can be mentioned at the start

[4 marks]

10.

6	(i)		$u_k = 5 + 1.5(k - 1)$ $5 + 1.5(k - 1) = 140$ $k = 91$	M1* M1d* A1 [3]	Attempt nth term of an AP, using $a = 5$ and $d = 1.5$ Equate to 140 and attempt to solve for k Obtain 91	Must be using correct formula, so M0 for $5 + 1.5k$ Allow if in terms of n not k Could attempt an nth term definition, giving $1.5k + 3.5$ Must be valid solution attempt, and go as far as an attempt at k Allow equiv informal methods Answer only gains full credit
	(ii)		$S_{16} = \frac{120(1-0.9^{16})}{1-0.9}$ $= 978$	M1 A1 [2]	Attempt to find the sum of 16 terms of GP, with $a = 120$, $r = 0.9$ Obtain 978, or better	Must be using correct formula If > 3sf, allow answer rounding to 977.6 with no errors seen Answer only, or listing and summing 16 terms, gains full credit

[5 marks]

11.

3(a)	$R = \sqrt{109}$	B1	1.1b
	$\tan\alpha = \frac{3}{10}$	M1	1.1b
	$\alpha = 16.70°$ so $\sqrt{109}\cos(\theta + 16.70°)$	A1	1.1b
		(3)	
(b)	(i) e.g $H = 11 - 10\cos(80t)° + 3\sin(80t)°$ or $H = 11 - \sqrt{109}\cos(80t + 16.70)°$	B1	3.3
	(ii) $11 + \sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t + "16.70" = 540$	M1	3.4
	$t = \frac{540 - "16.70"}{80} = (6.54)$	M1	1.1b
	$t = 6$ mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the '80' in the formula For example use $H = 11 - 10\cos(90t)° + 3\sin(90t)°$		3.3
		(1)	
		(9 marks)	

[9 marks]

12.

(i)		$P(A) \times P(B) = \frac{1}{24}$	M1	1.1a		
		$P(A) + P(B) = \frac{1}{24} + \frac{3}{8}$	M1	1.1		
		$P(A) + \frac{1}{24P(A)} = \frac{5}{12}$	M1	3.1a	Attempt equation in one P	
		$24(P(A))^2 - 10P(A) + 1 = 0$ $((6P(A) - 1)(4P(A) - 1) = 0)$	A1	1.1	Correct quadratic equation in one P	
		$P(A) = \frac{1}{6}$ and $P(B) = \frac{1}{4}$ or vice versa	A1	1.1	Allow without "vice versa"	
			[5]			
(ii)		$P(C') = 1 - P(C)$ $(= 0.4)$	M1	1.1		
		$P(D \cap C') = P(C \cup D) - P(C)$ $(= 0.2)$	M1	1.2		
		$P(D \mid C') = \frac{P(D \cap C')}{P(C')}$	M1	1.1	Attempted	
		$= \frac{0.2}{0.4} = 0.5$	A1	1.1		
			[4]			

[9 marks]