

**Write yours and your teacher's name at the top of your answer sheets.**

# **U6 Mathematics Mock**

## **Paper 2 (Y - COD/JW/SLJ)**

**February 2020**

**2019-2020**

**Duration: 1 hour 30 minutes**

**Total number of marks: 75**

***Write your answers on file paper.***

**You are permitted to use a scientific or graphical calculator in this paper.**

**Final answers should be given to a degree of accuracy appropriate to the context.**

1.

An arithmetic sequence has first term  $a$  and common difference  $d$ .

The sum of the first 16 terms of the sequence is 260

(a) Show that  $4a + 30d = 65$

[2 marks]

(b) Given that the sum of the first 60 terms is 315, find the sum of the first 41 terms.

[3 marks]

(c)  $S_n$  is the sum of the first  $n$  terms of the sequence.

Explain why the value you found in part (b) is the maximum value of  $S_n$

[2 marks]

[7 marks]

2.

Given that  $\theta$  is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$$

(3)

[3 marks]

3.

$$g(x) = \frac{2x + 5}{x - 3} \quad x \geq 5$$

(a) Find  $gg(5)$ .

(2)

(b) State the range of  $g$ .

(1)

(c) Find  $g^{-1}(x)$ , stating its domain.

(3)

[6 marks]

4.

Sketch the graph of  $y = |2x + a|$ , where  $a$  is a positive constant.

Show clearly where the graph intersects the axes.

[3 marks]

5.

(i) Express  $\frac{x+8}{x(x+2)}$  in partial fractions.

[3]

(ii) By first using division, express  $\frac{7x^2 + 16x + 16}{x(x+2)}$  in the form  $P + \frac{Q}{x} + \frac{R}{x+2}$ .

[3]

[3 marks]

6.

(a) Find the first three terms, in ascending powers of  $x$ , of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to  $\sqrt{2}$

Possible values of  $x$  that could be substituted into this expansion are:

- $x = -14$  because  $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{18}} = \frac{\sqrt{2}}{6}$
- $x = 2$  because  $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $x = -\frac{1}{2}$  because  $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{\frac{9}{2}}} = \frac{\sqrt{2}}{3}$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of  $x$  should not be used

(1)

(ii) state, giving a reason, which of the three values of  $x$  would lead to the most accurate approximation to  $\sqrt{2}$

(1)

[6 marks]

7.

The depth of water,  $D$  metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2 \sin(30t)^\circ \quad 0 \leq t < 24$$

where  $t$  is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.

The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)

[5 marks]

8.

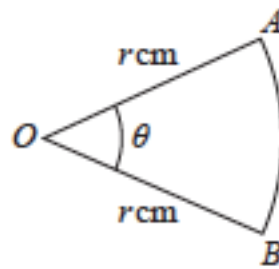


Figure 1

Figure 1 shows a sector  $AOB$  of a circle with centre  $O$  and radius  $r$  cm.

The angle  $AOB$  is  $\theta$  radians.

The area of the sector  $AOB$  is  $11 \text{ cm}^2$

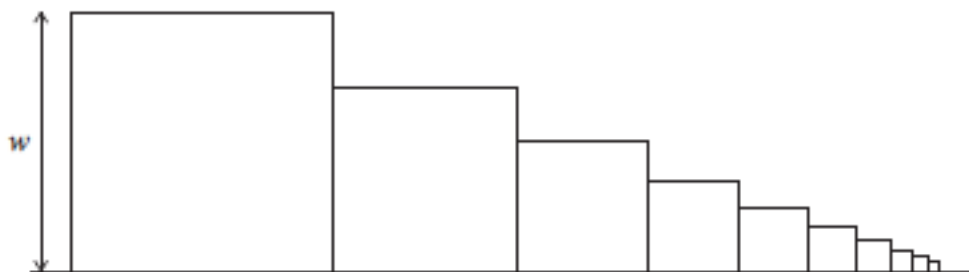
Given that the perimeter of the sector is 4 times the length of the arc  $AB$ , find the exact value of  $r$ .

(4)

[4 marks]

9.

Helen is creating a mosaic pattern by placing square tiles next to each other along a straight line.



The area of each tile is half the area of the previous tile, and the sides of the largest tile have length  $w$  centimetres.

- (a) Find, in terms of  $w$ , the length of the sides of the second largest tile.

[1 mark]

- (b) Assume the tiles are in contact with adjacent tiles, but do not overlap.

Show that, no matter how many tiles are in the pattern, the total length of the series of tiles will be less than  $3.5w$ .

[4 marks]

- (c) Helen decides the pattern will look better if she leaves a 3 millimetre gap between adjacent tiles.

Explain how you could refine the model used in part (b) to account for the 3 millimetre gap, and state how the total length of the series of tiles will be affected.

[2 marks]

[7 marks]

10.

(a) Prove that

$$1 - \cos 2\theta \equiv \tan \theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence solve, for  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ , the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3 \tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

[9 marks]

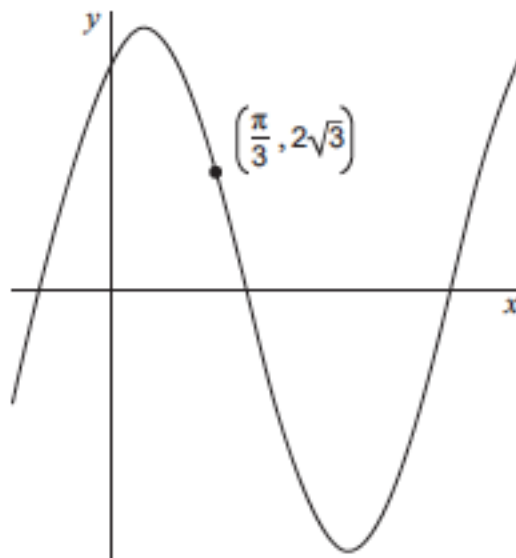
11.

A curve has equation

$$y = a \sin x + b \cos x$$

where  $a$  and  $b$  are constants.

The maximum value of  $y$  is 4 and the curve passes through the point  $(\frac{\pi}{3}, 2\sqrt{3})$  as shown in the diagram.



Find the exact values of  $a$  and  $b$ .

[6 marks]

[6 marks]

## Statistics

12.

A teacher in a college asks her mathematics students what other subjects they are studying.

She finds that, of her 24 students:

12 study physics  
8 study geography  
4 study geography and physics

- (a) A student is chosen at random from the class.

Determine whether the event 'the student studies physics' and the event 'the student studies geography' are independent.

**[2 marks]**

- (b) It is known that for the whole college:

the probability of a student studying mathematics is  $\frac{1}{5}$

the probability of a student studying biology is  $\frac{1}{6}$

the probability of a student studying biology given that they study mathematics is  $\frac{3}{8}$

Calculate the probability that a student studies mathematics or biology or both.

**[4 marks]**

[6 marks]

13.

Elizabeth's Bakery makes brownies.

It is known that the mass,  $X$  grams, of a brownie may be modelled by a normal distribution.

10% of the brownies have a mass less than 30 grams.

80% of the brownies have a mass greater than 32.5 grams.

Find the mean and standard deviation of  $X$ .

**[7 marks]**

[7 marks]