U6 Mock Teacher Y 19-20 SOLUTIONS [75]

1.

a)	Uses S_n =260 for arithmetic sequence with n=16 to form a correct equation PI by $8(2a+15d)=260$	1.1a	M1	$\frac{16}{2}(2a+(16-1)d)=260$ $8(2a+15d)=260$ $2(2a+15d)=65$ $4a+30d=65$
	Completes rigorous argument with correct algebraic manipulation to show required result **Must** see at least one line of simplification after $8(2a+15d)=260$ before given answer.	2.1	R1	
b)	Forms a second equation in *a* and *d* using $S_{60}=315$ and solves simultaneously to find *a* or *d*	3.1a	M1	$30(2a+59d)=315$ $20a+590d=105$ $a=20$ $d=-0.5$ $S_{41}=\frac{41}{2}(2\times20-40\times0.5)=410$
	Obtains correct *a* and *d*	1.1b	A1	
	Uses their *a* and *d* to obtain their value of $S_{41}=41a+820d$ Follow through provided one of their a or d is correct.	1.1b	A1F	
(c)	Explains that values of U_n are positive n < 41 Or Explains that values of U_n are negative for $n>41$ Or Uses quadratic manipulation or differentiation of formula for S_n to obtain n = 40.5 CSO	2.4	M1	The terms before the 41st term are all positive. The terms after the 41st term are all negative so the sum of the first 41 terms must be a maximum value.
	Completes a valid argument explaining all terms positive before 41 and negative after 41 Or Completes argument linking 40.5 with the sum to 40 terms and the sum to 41 terms. CSO	2.1	R1	
	Total		7	

2.

Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2} \rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
$= \frac{4}{3}$ oe	A1	1.1b
	(3)	

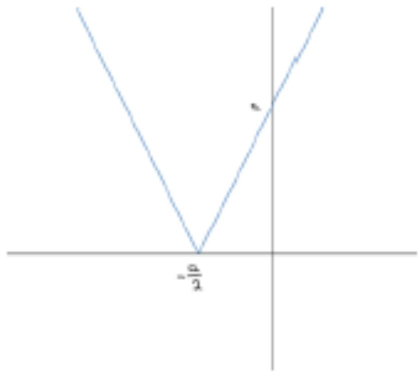
(3 marks)

3.

1	$g(x) = \frac{2x+5}{x-3},\ x \geq 5$		
(a) Way 1	$g(5) = \frac{2(5)+5}{5-3} = 7.5 \Rightarrow gg(5) = \frac{2("7.5")+5}{"7.5"-3}$	M1	1.1b
	$gg(5) = \frac{40}{9}\ \left(\text{or } 4\frac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(a) Way 2	$gg(x) = \frac{2\left(\frac{2x+5}{x-3}\right)+5}{\left(\frac{2x+5}{x-3}\right)-3} \Rightarrow gg(5) = \frac{2\left(\frac{2(5)+5}{(5)-3}\right)+5}{\left(\frac{2(5)+5}{(5)-3}\right)-3}$	M1	1.1b
	$gg(5) = \frac{40}{9}\ \left(\text{or } 4\frac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(b)	{Range:} $2 < y \leq \frac{15}{2}$	B1	1.1b
		(1)	
(c) Way 1	$y = \frac{2x+5}{x-3} \Rightarrow yx - 3y = 2x + 5 \Rightarrow yx - 2x = 3y + 5$	M1	1.1b
	$x(y-2) = 3y+5 \Rightarrow x = \frac{3y+5}{y-2}\ \left\{\text{or } y = \frac{3x+5}{x-2}\right\}$	M1	2.1
	$g^{-1}(x) = \frac{3x+5}{x-2},\quad 2 < x \leq \frac{15}{2}$	A1ft	2.5
		(3)	
(c) Way 2	$y = \frac{2x-6+11}{x-3} \Rightarrow y = 2 + \frac{11}{x-3} \Rightarrow y-2 = \frac{11}{x-3}$	M1	1.1b
	$x-3 = \frac{11}{y-2} \Rightarrow x = \frac{11}{y-2} + 3\ \left\{\text{or } y = \frac{11}{x-2} + 3\right\}$	M1	2.1
	$g^{-1}(x) = \frac{11}{x-2} + 3,\quad 2 < x \leq \frac{15}{2}$	A1ft	2.5
		(3)	

(6 marks)

4.

Draws the correct V-shape, nothing below x-axis	AO1.2	M1	
Intersects negative x-axis with $-\frac{a}{2}$ labelled	AO1.1b	A1	
Intersects positive y-axis with a labelled	AO1.1b	A1	
Total		**3**	

5.

(i)	$\frac{A}{x}+\frac{B}{x+2}$	**B1**		award if only implied by answer
	$x+8=A(x+2)+Bx$ soi	**M1**	allow one sign error	clearing fractions successfully
	$A=4$ *and* $B=-3$	**A1**		if **M0**, **B1** for each value www
		[3]		
(ii)	quotient (P) is 7	**B1**		
	$2x+16$ seen	**B1**	if **B0**, **B1** for $Q=8$ and **B1** for $R=-6$ www	eg as remainder or in division chunking
	$7+\frac{8}{x}-\frac{6}{x+2}$	**B1**		or allow $P=7, Q=8$ $R=-6$
		[3]		

6.

4 (a)	$\frac{1}{\sqrt{4-x}}=(4-x)^{-\frac{1}{2}}=4^{-\frac{1}{2}}\times(1\pm......$	M1	2.1
	Uses a "correct" binomial expansion for their $(1+ax)^n=1+nax+\frac{n(n-1)}{2}a^2x^2+$	M1	1.1b
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$	A1	1.1b
	$\frac{1}{\sqrt{4-x}}=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	A1	1.1b
		(4)	
(b) (i)	States $x=-14$ and gives a valid reason. Eg explains that the expansion is not valid for $\lvert x\rvert>4$	B1	2.4
		(1)	
(b)(ii)	States $x=-\frac{1}{2}$ and gives a valid reason. Eg. explains that it is closest to zero	B1	2.4
		(1)	
			(6 marks)

7.

(a)	$D=5+2\sin(30\times6.5)°=$ awrt 4.48m with units	B1	3.4
		(1)	
(b)	$3.8=5+2\sin(30t)°\Rightarrow\sin(30t)°=-0.6$	M1 A1	1.1b 1.1b
	$t=10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

8.

States or uses $\frac{1}{2}r^2\theta=11$	B1	1.1b
States or uses $2r+r\theta=4r\theta$	B1	1.1b
Attempts to solve, full method $r=...$	M1	3.1a
$r=\sqrt{33}$	A1	1.1b
		[4]
		(4 marks)

9.

(a)	Obtains correct length $\frac{w}{\sqrt{2}}=\frac{\sqrt{2}w}{2}$ ACF	AO1.1b	B1	$\frac{w}{\sqrt{2}}$
(b)	Models the lengths as a geometric sequence	AO3.3	M1	$a=w$ and $r=\frac{1}{\sqrt{2}}$ $S_\infty=\frac{w}{1-\frac{1}{\sqrt{2}}}$ $\approx 3.41w<3.5w$
	Finds the sum to infinity provided their $r<1$	AO1.1a	M1	
	Uses their model to obtain the correct sum in terms of w	AO3.4	A1	
	Compares their sum with 3.5w	AO2.4	E1	
(c)	Explains that the model would have to include an additional 3 mm for each tile	AO3.5c	E1	The total length will now include an additional 3 mm for each tile. The total length will not have an upper limit.
	Explains that the total length will not have an upper limit Or The total length may now exceed 3.5w	AO3.5a	E1	
	Total		**7**	

10.

12	$1-\cos 2\theta \equiv \tan\theta \sin 2\theta,\ \theta \neq \frac{(2n+1)\pi}{2},\ n\in\mathbb{Z}$		
(a) Way 1	$\tan\theta\sin 2\theta = \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$	M1	1.1b
	$=\left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$	M1	1.1b
	$=\left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x-5)(1-\cos 2x) = 3\tan^2 x\sin 2x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$		
(b) Way 1	$(\sec^2 x-5)\tan x\sin 2x = 3\tan^2 x\sin 2x$ **or** $(\sec^2 x-5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$		
	Deduces $x=0$	B1	2.2a
	Uses $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ e.g. $(1+\tan^2 x-3\tan x-5)\tan x=0$ **or** $(1+\tan^2 x-3\tan x-5)(1-\cos 2x)=0$ **or** $1+\tan^2 x-5=3\tan x$	M1	2.1
	$\tan^2 x-3\tan x-4=0$	A1	1.1b
	$(\tan x-4)(\tan x+1)=0 \Rightarrow \tan x = ...$	M1	1.1b
	$x=-\frac{\pi}{4}, 1.326$	A1	1.1b
		A1	1.1b
		(6)	
			(9 marks)

11.

Compares with $R\cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$ by forming an identity e.g. $R\sin(x+\alpha) \equiv a\sin x + b\cos x$ OE or Differentiates correctly and equates to zero CAO **PI** by $a\cos x = b\sin x$ **PI** by $R = 4$ or $a^2 + b^2 = 16$	3.1a	M1	$R\sin(x+\alpha) = a\sin x + b\cos x$ $R = 4$ $4\sin\left(\frac{\pi}{3}+\alpha\right) = 2\sqrt{3}$ $\alpha = \frac{\pi}{3}$ $a = 4\cos\frac{\pi}{3} = 2$ $b = 4\sin\frac{\pi}{3} = 2\sqrt{3}$
Deduces $R = 4$ or $a^2 + b^2 = 16$	2.2a	A1	
Forms a correct equation for α **PI** by correct α or Forms the equation shown below $2\sqrt{3} = \frac{a\sqrt{3}}{2} + \frac{b}{2}$ OE **Must** substitute correct exact values for the trig functions	1.1b	B1	
Solves their equation to obtain any correct value of α Correct values are shown below $\alpha = \frac{\pi}{3}$ or 0 for $R\sin(x \pm \alpha)$ $\alpha = \pm\frac{\pi}{6}$ for $R\cos(x \pm \alpha)$ or Eliminates a variable correctly from their two equations – **must** obtain a correct simplified equation	1.1a	M1	
Deduces $a = 2$	2.2a	R1	
Deduces $b = 2\sqrt{3}$	2.2a	R1	
Total		**6**	

[9 marks]

12.

(a)	Calculates $P(\text{studies Physics}) \times P(\text{studies Geography})$ or Calculates $P(\text{studies Geography} \mid \text{studies Physics})$ or $P(\text{studies Physics} \mid \text{studies Geography})$	AO3.1b	M1	$P(P) \times P(G) = \frac{12}{24} \times \frac{8}{24} = \frac{1}{6}$
	Shows $P(\text{studies Physics}) \times P(\text{studies Geography})$ $= P(\text{studies Physics} \cap \text{studies Geography})$ and correctly concludes that the events are independent or Shows that the appropriate conditional probability is equal to $P(\text{studies Geography})$ or $P(\text{studies Physics})$ and correctly concludes that the events are independent	AO2.1	R1	$P(P \cap G) = \frac{4}{24} = \frac{1}{6}$ Hence $P(P) \times P(G) = P(P \cap G)$ Therefore events are independent
(b)	Uses conditional probability to calculate $P(M \cap B)$	AO3.1b	M1	$P(M \cap B) = P(M) \times P(B \mid M)$ $= \frac{1}{5} \times \frac{3}{8} = \frac{3}{40}$ $P(M) + P(B) - P(M \cap B)$ $= \frac{1}{5} + \frac{1}{6} - \frac{3}{40}$ $= \frac{7}{24}$
	Obtains the correct value of $P(M \cap B)$	AO1.1b	A1	
	Uses the addition rule to calculate $P(M \cup B)$	AO1.1a	M1	
	Obtains the correct value of $P(M \cup B)$	AO1.1b	A1	
	Total		**6**	

13.

Obtains either z-value from inverse normal distribution Condone sign error **AWFW** [−1.29, −1.28] or [−0.85, −0.84]	3.1b	B1	$P\left(Z < \frac{30-\mu}{\sigma}\right) = 0.1$ $P\left(Z > \frac{32.5-\mu}{\sigma}\right) = 0.8$ z= −1.2816 z = −0.8416 $\frac{30-\mu}{\sigma} = -1.2816$ $\frac{32.5-\mu}{\sigma} = -0.8416$ $2.5 = 0.44\sigma$ $\sigma = 5.68$ $\mu = 37.3$
Forms one equation with unknown μ and σ using standardised result and z-value (for 0.1) Accept z = (−4, 4) except ±0.1, ±0.2, ±0.8, ±0.9 Condone $\mu - 30$ Must use 30	1.1a	M1	
Forms next equation with unknown μ and σ using standardised result and z-value (for 0.8) Accept z = (−4, 4) except ±0.1, ±0.2, ±0.8, ±0.9 Condone $\mu - 32.5$ Must use 32.5	1.1a	M1	
Obtains both equations correctly	1.1b	A1	
Solves their two simultaneous equations in the form of μ and σ	1.1a	M1	
Obtains correct value of σ **AWFW** (5.2, 5.9) **ISW**	1.1b	A1	
Obtains correct value of μ **AWFW** (37.1, 37.5) **ISW**	1.1b	A1	