Arcs and sectors

Starter

1. Given that $1^c = \frac{180^\circ}{\pi} \approx 57.3^\circ$:

   (a) how do we convert from radians to degrees?
   (b) how do we convert from degrees to radians?

   Working: (a) $\times \frac{180}{\pi}$

   (b) $\times \frac{\pi}{180}$

2. Convert these angles to radians: (a) 45° (b) 60°

   Working: (a) $45^\circ \times \frac{\pi}{180} = \frac{\pi}{4}$

   (b) $60^\circ \times \frac{\pi}{180} = \frac{\pi}{3}$

3. Convert these angles to degrees: (a) $\frac{\pi}{6}$ (b) 2π.

   Working: (a) $\frac{\pi}{6} \times \frac{180}{\pi} = 30^\circ$

   (b) $2\pi \times \frac{180}{\pi} = 360^\circ$

E.g. 1 From GCSE, the length of arc, s, for a sector whose radius is r and whose angle subtended at the centre is θ°, where θ is measured in degrees is given by the formula:

$$s = \frac{\theta^\circ}{360^\circ} \times 2\pi r$$

By converting the angle to radians, find a formula for the length of arc when the angle is given in radians.

Working: Degrees to radians: $\times \frac{\pi}{180}$ $\Rightarrow$ $\theta^\circ \times \frac{\pi}{180} = \theta^c$

$$\theta^\circ = \theta^c \times \frac{180}{\pi}$$

Substitute in $s = \frac{\theta^\circ}{360^\circ} \times 2\pi r$:

$$s = \frac{\theta^c \times \frac{180}{\pi}}{360^\circ} \times 2\pi r$$

$$s = \frac{180\theta^c}{\pi 360^\circ} \times 2\pi r$$

$$s = r\theta^c$$

E.g. 2 Using a similar method to ***E.g. 1*** and utilising the formula $A = \frac{\theta^\circ}{360^\circ} \times \pi r^2$, find a formula for area of sector, when the angle is measured in radians.

Working: $\theta^\circ = \theta^c \times \frac{180}{\pi}$

Substitute in $A = \frac{\theta^\circ}{360^\circ} \times \pi r^2$: $A = \frac{\theta^c \times \frac{180}{\pi}}{360^\circ} \times \pi r^2$

$$A = \frac{180\theta^c}{\pi 360^\circ} \times \pi r^2$$

$$A = \frac{1}{2} r^2 \theta^c$$

E.g. 3 Find the radius of the sector whose arc length is 8.5 cm and where the angle subtended at the centre is $\frac{3\pi}{4}$.

Working: Using $s = r\theta^c$: $8.5 = r \times \frac{3\pi}{4} \quad \Rightarrow \quad r = \frac{34}{3}$ cm

The radius of the sector is $\frac{34}{3\pi} \approx 3.61$ cm

E.g. 4 Find the area of a sector whose radius is 5 cm and where the angle subtended at the centre is $\frac{2\pi}{5}$.

Working: Using $A = \frac{1}{2} r^2 \theta^c$: $A = \frac{1}{2} \times 5^2 \times \frac{2\pi}{5} = 5\pi$

The area of a sector is 5π cm².

E.g. 5 Find the perimeter of the sector whose angle subtended at the centre is $\frac{5\pi}{8}$ and whose area is 20π cm². Give your answer in terms of π.

Working: Using $A = \frac{1}{2} r^2 \theta^c$: $20\pi = \frac{1}{2} \times r^2 \times \frac{5\pi}{8}$

$$r^2 = 64$$

$$r = 8$$

Using $s = r\theta^c$: $s = 8 \times \frac{5\pi}{8} = 5\pi$

The perimeter of the sector is $(5\pi + 16)$ cm

Video: Arcs, sectors and segments

Arcs, sectors and segments EQ

Solutions to Starter and E.g.s

Exercise

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