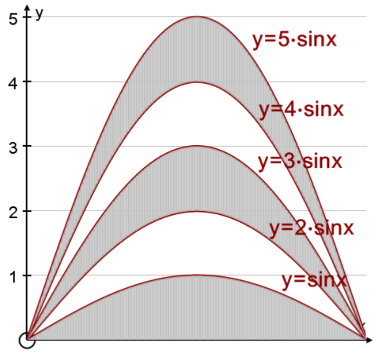
# Areas between Curves and the $y-$axis

**Starter**

1. **(Review of last lesson)** A company has designed a logo based on multiples of the sine curve as shown. Calculate the total area of the grey sections of the logo.




***Working:*** Lowest area $= \int_0^{\pi} \sin x dx$

$$= \left[-\cos x\right]_0^{\pi}$$

$$= \left[\cos x\right]_{\pi}^{0}$$

$$= 1 - -1 = 2$$

Middle area $= \int_0^{\pi} 3\sin x dx - \int_0^{\pi} 2\sin x dx = \int_0^{\pi} \sin x dx = 2$

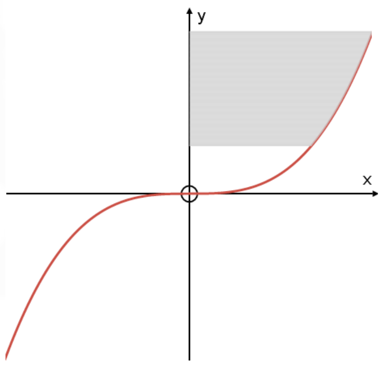
Top area $= \int_0^{\pi} 5\sin x dx - \int_0^{\pi} 4\sin x dx = \int_0^{\pi} \sin x dx = 2$

Total grey area $= 6$

2. If the area between a curve and the $x-$axis is $\int_{x_1}^{x_2} y dx$, write down a formula for the area between a curve and the $y-$axis.

***Working:*** Area between curve and the $y-$axis $= \int_{y_1}^{y_2} x dy$, where $y_1$ and $y_2$ are the $y-$ values for the limits.

3. Find the area of the region bounded by parts of the $y-$axis, the curve $y = x^3$ and the lines $y = 8$ and $y = 27$.




***Working:*** Rearrange $y = x^3$ to make $x$ the subject:
$x = y^{\frac{1}{3}}$

Required area $= \int_8^{27} y^{\frac{1}{3}} dy$

$$= \left[\frac{3}{4}y^{\frac{4}{3}}\right]_8^{27}$$

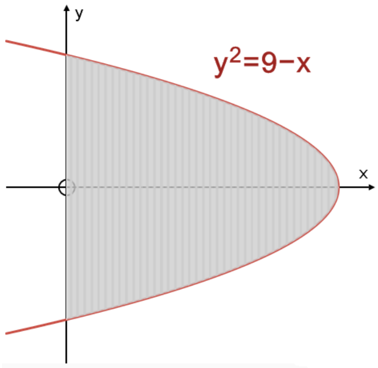
$$= \left(\frac{3}{4} \times 27^{\frac{4}{3}}\right) - \left(\frac{3}{4} \times 8^{\frac{4}{3}}\right)$$

$$= \left(\frac{3}{4} \times 81\right) - \left(\frac{3}{4} \times 16\right)$$

$$= \frac{243}{4} - 12$$

$$= \frac{195}{4} = 48.75$$

***E.g. 1*** The graph shows part of the graph of $y^2 = 9 - x$ for which $x > 0$. Find the area between this curve and the $y-$axis.




***Working:*** To find the limits find the points where the curve $y^2 = 9 - x$ meets the $y-$axis:

$y-$axis $\equiv x = 0$

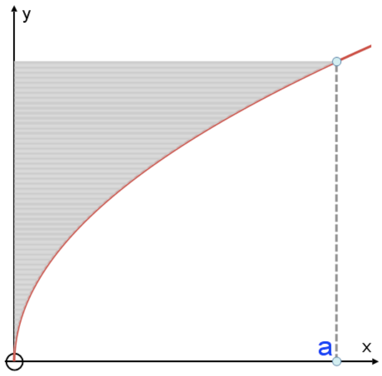
Solve $y^2 = 9 \quad \Rightarrow \quad y = \pm 3$

Rearrange $y^2 = 9 - x$ to make $x$ the subject: $x = 9 - y^2$

$$\text{Shaded area} = \int_{-3}^{3} (9 - y^2)dy$$
$$= \left[9y - \frac{1}{3}y^3\right]_{-3}^{3}$$
$$= \left(27 - \frac{1}{3} \times 27\right) - \left(-27 - \frac{1}{3} \times (-27)\right)$$
$$= 36$$

***N.B.*** The area could also be found by doubling the area from $y = 0$ to $y = 3$ i.e. $\int_{-3}^{3} (9 - y^2)dy = 2\int_{0}^{3} (9 - y^2)dy$

***E.g. 2*** The diagram shows the graph of $y = \sqrt{x}$. Given that the area of the shaded region is 72, find the value of $a$.




***Working:*** When $x = a$, $y = \sqrt{a}$

$y = \sqrt{x} \quad \Rightarrow \quad x = y^2$

$$\text{Shaded area} = \int_{0}^{\sqrt{a}} y^2 dy$$
$$= \left[\frac{1}{3}y^3\right]_{0}^{\sqrt{a}}$$
$$= \left(\frac{1}{3} \times (\sqrt{a})^3\right) - (0)$$

Since the shaded area is 72:
$$\frac{1}{3} \times (\sqrt{a})^3 = 72$$
$$(\sqrt{a})^3 = 216$$
$$\sqrt{a} = 6$$
$$a = 36$$

Video: Area between curve and y-axis

Solutions to Starter and E.g.s

**Exercise**
p275 Ex 12F Qu 2, 7, 8, 12