Arithmetic Sequences

Starter

1. **(Review of last lesson)** Write down the terms of $\sum_{r=-2}^{3}(r^2+1)$ and hence find its value.

Working: $\sum_{r=-2}^{3}(r^2+1) = ((-2)^2+1)+((-1)^2+1)+(0^2+1)+(1^2+1)+(2^2+1)+(3^2+1)$

$= 5+2+1+2+5+10 = 25$

2. **(Review of last lesson)** Express the series $-25 + 36 - 49 + 64$ in sigma notation.

Working: 25, 36, 49, 64 are the square numbers so r^2
Signs go $-$ $+$ $-$ $+$… so even terms are positive $\Rightarrow$ $(-1)^r$

$\sum_{r=5}^{8}(-1)^r r^2$ oe

N.B. oe is mark scheme notation and means "or equivalent" (i.e. equivalent answers are accepted)

E.g. 1 (a) Given that the first term of a general AP is a and the common difference is d, write down the first 4 terms.
(b) Hence express the the $n-$th term, u_n, in terms of a, d and n.

Working: (a) $a, a+d, a+2d, a+3d, \ldots.$
(b) The coefficient of d is always 1 less than then "th term"
i.e. 2nd term has $1d$, 3rd term has $2d$ etc.
Formula for $n-$th term is $u_n = a+(n-1)d$

E.g. 2 Find the formula for the $n-$th term for the sequence 13, 15, 17, 19,…

Working: $a = 13$ and $d = 15-13 = 2$
$u_n = a+(n-1)d \Rightarrow u_n = 13+(n-1)\times 2 = 2n+11$

E.g. 3 Find the number of terms in the sequence 10, 7, 4, …, -50.

Working: $a = 10$ and $d = 7-10 = -3$
$u_n = a+(n-1)d \Rightarrow u_n = 10+(n-1)\times(-3) = 13-3n$
-50 is the last term so $13-3n = -50 \Rightarrow 63 = 3n \therefore n = 21$
There are 21 terms in the sequence.

E.g. 4 The 5th term of an AP is 7 and the 9th term is 43. Find the first term and the common difference.

Working: $u_n = a+(n-1)d$: $u_5 = 7 \Rightarrow a+4d = 7$
$u_9 = 43 \Rightarrow a+8d = 43$
Solving simultaneously gives $a = -29, d = 9$
1st term = -29, common difference = 9

E.g. 5* In an arithmetic sequence the first 3 terms are $\ln x, \ln(x+8)$ and $\ln(x+48)$. Find the value of x and the next term in the sequence.

Hint: You will need the laws of logs.

Working: $\ln x, \ln(x+8)$ and $\ln(x+48)$

$d = u_2 - u_1 = \ln(x+8) - \ln x = \ln\left(\frac{x+8}{x}\right)$

Also $d = u_3 - u_2 = \ln(x+48) - \ln(x+8) = \ln\left(\frac{x+48}{x+8}\right)$

$\ln\left(\frac{x+48}{x+8}\right) = \ln\left(\frac{x+8}{x}\right) \quad \Rightarrow \quad \left(\frac{x+48}{x+8}\right) = \left(\frac{x+8}{x}\right)$

Cross multiply: $x(x+48) = (x+8)^2$

Expand: $x^2 + 48x = x^2 + 16x + 64$

Collect like terms: $32x = 64$

$\therefore x = 2,$

So $d = \ln\left(\frac{2+8}{2}\right) = \ln 5$

The next term is $\ln(2+48) + \ln 5 = \ln 50 + \ln 5 = \ln 250$

Video: Arithmetic progressions

Video: Finding a and d given two terms

Solutions to Starter and E.g.s

Exercise

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