Arithmetic Series

Starter

1. **(Review of last lesson)**
   Find the number of terms in the sequence 72, 64, 56, 48, …, −288.

   Working: $a = 72$ and $d = 64 - 72 = -8$
   $u_n = a + (n-1)d \quad \Rightarrow \quad u_n = 72 + (n-1)\times(-8) = 80 - 8n$
   −288 is the last term so $80 - 8n = -288 \quad \Rightarrow \quad 368 = 8n$
   $\therefore n = 46$
   There are 46 terms in the sequence

2. **(Review of last lesson)** In an arithmetic progression, $u_3 = 15$ and $u_{26} = 84$. Find the value of k if find $u_k = 66$.

   Working: $u_n = a + (n-1)d$: $\quad u_3 = 15 \quad \Rightarrow \quad a + 2d = 15$
   $u_9 = 43 \quad \Rightarrow \quad a + 25d = 84$
   Solving simultaneously gives $a = 9, d = 3$
   $u_k = 66 \quad \Rightarrow \quad 9 + (k-1)\times 3 = 66 \quad \Rightarrow \quad 3k + 6 = 66$
   $\therefore k = 20$

3. Find the sum of the first 100 positive integers

   Working: Let $S = 1 + 2 + 3 + \ldots + 98 + 99 + 100$
   Also $S = 100 + 99 + 98 + \ldots + 3 + 2 + 1$
   Adding the two series together: $\quad 2S = 101 + 101 + 101 + \ldots + 101$
   $2S = 100 \times 101 = 10100 \quad \Rightarrow \quad S = 5050$
   $1 + 2 + 3 + 4 + \ldots + 100 = 5050$

E.g. 1 Employ the same method used in question 3 of the starter find the sum of the series:

$$S_n = a \; + \; a + d \; + \; a + 2d \; + \; \ldots \; + \; a + (n-3)d \; + \; a + (n-2)d \; + \; a + (n-1)d$$

Hint: Write the series out twice — once with the terms in the right order and the 2nd time with the order of the terms reversed.

Working:
Write the series out twice

$$S_n = a \; + \; a + d \; + \; a + 2d \; + \; \ldots \; + \; a + (n-3)d \; + \; a + (n-2)d \; + \; a + (n-1)d$$

$$S_n = a + (n-1)d \; + \; a + (n-2)d \; + \; a + (n-3)d \; + \; \ldots \; + \; a + 2d \; + \; a + d \; + \; a$$

The sum of the first term and the last is equal to the sum of the second and the penultimate terms etc.

Adding the two series together gives

$$2S_n = 2a + (n-1)d \; + \; 2a + (n-1)d \; + \; 2a + (n-1)d \; + \; \ldots \; + \; 2a + (n-1)d$$

There n terms, each of $2a + (n-1)d$.

$$2S_n = n(2a + (n-1)d)$$

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ — the formula for the sum of the 1st n terms of an arithmetic progression

E.g. 2 Find the sum of the 1st 14 terms of 2 + 5 + 8 + …

Working: $a = 2, d = 5 - 2 = 3, n = 14$

Using $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$: $S_n = \frac{14}{2}\left(2 \times 2 + (13-1)3\right) = 301$

E.g. 3 The 7th term of an AP is −14 and the sum of the first 7 terms is −35. Find the first term and the common difference.

Working:

$a_7 = -14$: $a + (7-1)d = -14 \Rightarrow a + 6d = -14$

$S_7 = -14$: $\frac{7}{2}(2a + 6d) = -35 \Rightarrow 2a + 6d = -10$

Solving simultaneously gives $a = 4, d = -3$

E.g. 4 How many terms are needed for the sum of the series 1 + 5 + 9 + … to equal 231?

Working: $a = 1, d = 5 - 1 = 4$

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$: $\frac{n}{2}\left(2 + (n-1) \times 4\right) = 231$

$n(4n - 2) = 462 \Rightarrow 4n^2 - 2n - 462 = 0$

Solving $2n^2 - n - 231 = 0$ gives $n = 11$ or $n = -10.5$

Since n must be a positive integer, there are 11 terms in the sequence.

E.g. 5 Given that the last term of a sequence is $l = a + (n-1)d$, write S_n in terms of a, l and n.

Working: $S_n = \frac{n}{2}\left(2a + (n-1)d\right) = \frac{n}{2}\left(a + a + (n-1)d\right)$

Since $l = a + (n-1)d \Rightarrow S_n = \frac{n}{2}(a + l)$

Video: Sum of arithmetic series

Arithmetic progressions EQ

Solutions to Starter and E.g.s

Exercise

p74 4D Qu 1i, 2i, 3-16