Chain Rule (brackets)

Starter

1. **(Review of last lesson)** Find the area between the $x-$axis and the curve $y = \cos x$ between $x = 0$ and $x = \frac{2\pi}{3}$.

Hint: Draw a sketch to help.

Working: Be careful — part of the area is below the $x-$axis so we need to calculated separate integrations and make the area below the $x-$axis positive.

$$\text{Area} = \int_0^{\frac{\pi}{2}} \cos x\,dx + \left|\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \cos x\,dx\right| = 1 + \left|\frac{\sqrt{3}}{2} - 1\right| = 2 - \frac{\sqrt{3}}{2}$$

2. Find $\frac{dy}{dx}$ when: (a) $y = (3x-4)^2$ (b) $y = (3x-4)^{17}$

N.B. Don't spend too long on (b) if you are having difficulty…

Working: (a) Expand the brackets: $y = (3x-4)^2 = 9x^2 - 24x + 16$

Then differentiate: $\frac{dy}{dx} = 18x - 24 = 6(3x-4)$

(b) Takes too long due to do the expansion, there must be a quicker way.

E.g. Differentiate $y = (3x-4)^{17}$.

Working: Let $u = 3x - 4$ so $\frac{du}{dx} = 3$

$y = u^{17}$ so $\frac{dy}{du} = 17u^{16}$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = 3 \times 17u^{16}$$

Replace u by $3x-4$ and simplify: $\frac{dy}{dx} = 51(3x-4)^{16}$

E.g. Differentiate $y = (7-4x)^6$.

Working: Let $u = 7 - 4x$ so $\frac{du}{dx} = -4$

$y = u^6$ so $\frac{dy}{du} = 6u^5$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = -4 \times 6u^5$$

Replace u by $7-4x$ and simplify: $\frac{dy}{dx} = -24(7-4x)^5$

E.g. 1 Differentiate: (a) $y = (5x+2)^8$ (b) $y = (x^2 + 3x - 9)^5$

(c) $f(x) = \sin^2 x$ (d) $y = (ax+b)^n$

Hint: For (c), rewrite $f(x) = \sin^2 x$ as $f(x) = (\sin x)^2$.

Working: (a) Let $u = 5x + 2$ so $\dfrac{du}{dx} = 5$

$y = u^8$ so $\dfrac{dy}{du} = 8u^7$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = 5 \times 8u^7$$

Replace u by $5x + 2$ and simplify: $\dfrac{dy}{dx} = 40(5x + 2)^7$

(b) Let $u = x^2 + 3x - 9$ so $\dfrac{du}{dx} = 2x + 3$

$y = u^5$ so $\dfrac{dy}{du} = 5u^4$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = (2x + 3) \times 5u^4$$

Replace u by $x^2 + 3x - 9$: $\dfrac{dy}{dx} = 5(2x + 3)(x^2 + 3x - 9)^4$

(c) Let $u = \sin x$ so $\dfrac{du}{dx} = \cos x$

$y = u^2$ so $\dfrac{dy}{du} = 2u$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = \cos x \times 2u$$

Replace u by $\sin x$: $\dfrac{dy}{dx} = 2\cos x \sin x$

(d) Let $u = ax + b$ so $\dfrac{du}{dx} = a$

$y = u^n$ so $\dfrac{dy}{du} = nu^{n-1}$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = a \times nu^{n-1}$$

Replace u by $ax + b$: $\dfrac{dy}{dx} = an(ax + b)^{n-1}$

E.g. 2 Differentiate $y = \left[f(x)\right]^n$

Working: Let $u = f(x)$ so $\dfrac{du}{dx} = f'(x)$

$y = u^n$ so $\dfrac{dy}{du} = nu^{n-1}$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = f'(x) \times nu^{n-1}$$

Replace u by $f(x)$: $\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$

The power is reduced by 1

The power comes down to the front

Multiply by the derivative of the bracket

In general: $y = \left[f(x)\right]^n \quad \Rightarrow \quad \dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$

E.g. Differentiate $y = (x^2 - 7)^4$.

Working: $\dfrac{dy}{dx} = 4 \times 2x \times (x^2 - 9)^3 = 8x(x^2 - 9)^3$

$n \quad f'(x) \quad \left[f(x)\right]^{n-1}$

E.g. 3 Without using "Let $u = \ldots$", differentiate $y = 6(x^3 - 5x)^{11}$

Working: $\dfrac{dy}{dx} = 6 \times 11 \times (3x^2 - 5) \times (x^3 - 5x)^{10} = 66(3x^2 - 5)(x^3 - 5x)^{10}$

$n \qquad f'(x) \qquad \left[f(x)\right]^{n-1}$

Video: Chain rule (brackets)

Solutions to Starter and E.g.s

Exercise
p201 10A Qu 1iabc, 2iagh, 3iab