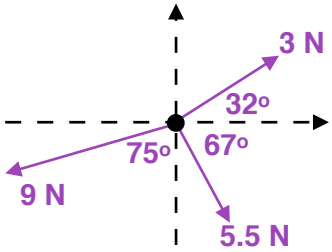
# Coefficient of Friction

**Starter**

1. **(Review of last lesson)** Find the magnitude and direction of the resultant force acting on the particle.




***Working:*** $R(\rightarrow)$: $3\cos 32 + 5.5\cos 67 - 9\sin 75 \approx -4.000$

$R(\uparrow)$: $3\sin 32 - 5.5\sin 67 - 9\cos 75 \approx -5.802$

By Pythagoras, the resultant, $R = \sqrt{4.000^2 + 5.802^2} = 7.05$ N (3 s.f.)

***The resultant force is in the 3rd quadrant***

The angle is $180 + \tan^{-1}\dfrac{5.802}{4.000} = 235.4^{\circ}$ (3 s.f.)

2. **(Review of last lesson)** A trolley is on a level, smooth floor. A girl is trying to push it with a force of $10$ N at $15^{\circ}$ to the horizontal. Her brother prevents it from moving by pushing horizontally with a force in the opposite direction. With what force does he push?

***Working:*** $R(\rightarrow)$: $P = 10\cos 15 = 9.66$ N

***N.B.*** The mass of the trolley is unimportant.

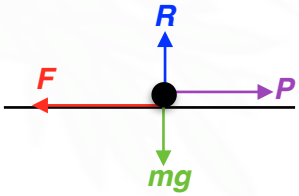
***E.g. 1*** A horizontal force $P$ newtons is applied to a body of weight 80 N, standing in rough contact with a horizontal plane. The coefficient of friction between the body and the plane is 0.5.
What is the magnitude of the frictional force, $F$, when:

(a) $P = 10$ N

(b) $P = 40$ N

(c) $P = 50$ N?

State in each case whether or not the body moves.




***Working:*** (a) $P = 10$ N, the body does not move

(b) $F = 40$ N, the body is on the point of moving

(c) $F = 40$ N, th body moves

***E.g. 2*** A particle of mass $m$ kg lies on a rough horizontal plane. The coefficient of friction between the particle and the place is $\mu$. A force of $P$ N acts at an angle $\theta$ above the horizontal. For the situations where the particle is:

(a) accelerating and

(b) in limiting equilibrium or moving with constant speed:

(i) Resolve vertically, $R(\uparrow)$

(ii) Use $F_{lim} = \mu R$

(iii) Use $F = ma(\rightarrow)$ or resolve horizontally $R(\rightarrow)$

***Working:***

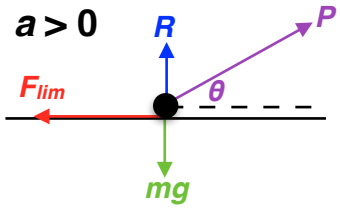
(a) ***Accelerating***

(i) $R(\uparrow)$: $R + P\sin\theta = mg$

$R = mg - P\sin\theta$

(ii) $F_{lim} = \mu R$: $F_{lim} = \mu(mg - P\sin\theta)$

(iii) $F = ma(\rightarrow)$: $P\cos\theta - F_{lim} = ma$



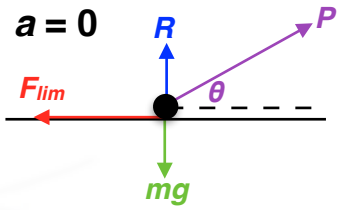

(b) ***Stationary or moving with constant speed***

(i) $R(\uparrow)$: $R + P\sin\theta = mg$

$R = mg - P\sin\theta$

(ii) $F_{lim} = \mu R$: $F_{lim} = \mu(mg - P\sin\theta)$

(iii) $R(\rightarrow)$: $P\cos\theta = F_{lim}$




***E.g. 3*** A small block of weight $32$ N is lying in rough contact on a horizontal plane. A horizontal force of P newtons is applied to the block until it is just about to move the block.

(a) If $P = 8$, find the coefficient of friction, $\mu$, between the block and the plane.

(b) If $\mu = 0.4$, find the value of P.

***Working:***

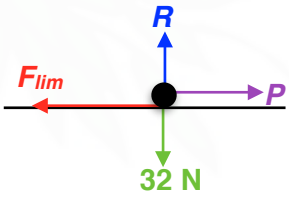
(a) $P = 8$

$R(\uparrow)$: $R = 32$

$R(\rightarrow)$: $F_{lim} = P = 8$

$F_{lim} = \mu R$: $8 = 32\mu$

$\mu = 0.25$




(b) $\mu = 0.4$

$R(\uparrow)$: $R = 32$

$F_{lim} = \mu R$: $F_{lim} = 0.4 \times 32 = 12.8$

$R(\rightarrow)$: $P = F_{lim} = 12.8$ N

***E.g. 4*** A particle of mass $5$ kg, resting on a rough plane, is acted on by a force of $23$ N. The coefficient of friction between the particle and surface is $\frac{1}{3}$.

(a) Find the acceleration when the force acts horizontally.
(b) If the force acts at an angle of $20^{\circ}$ to the horizontal will the acceleration increase or decrease.
(c) Find the new acceleration.

***Working:***

(a) $\mu = \frac{1}{3}$

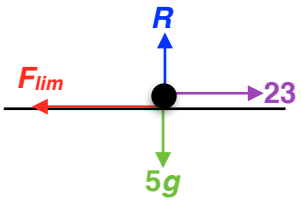
$R(\uparrow)$: $R = 5g$

$F_{lim} = \mu R$: $F_{lim} = \frac{1}{3} \times 5g = \frac{5}{3}g$

$F = ma(\rightarrow)$: $23 - F_{lim} = 5a$

$23 - \frac{5}{3}g = 5a$

$a = \frac{4}{3}$ m/s²




(b) Check your answer after doing (c).

(c) $R(\uparrow)$: $R + 23\sin 20 = 5g$

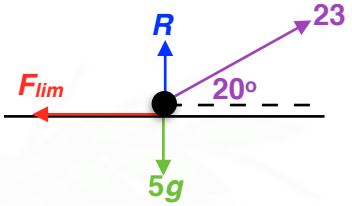
$R = 5g - 23\sin 20$

$F_{lim} = \mu R$: $F_{lim} = \frac{1}{3}(5g - 23\sin 20)$

$F = ma(\rightarrow)$: $23\cos 20 - F_{lim} = 5a$

$23 - \frac{1}{3}(5g - 23\sin 20) = 5a$

$a = 1.58$ m/s²




***N.B.*** As can be seen the acceleration increases when the force acts at $20^{\circ}$ to the horizontal.

***E.g. 5*** A particle of mass $12$ kg is being pulled along a rough surface by a horizontal force of $P$ N such that its acceleration is $3$ m/s$^2$. The coefficient of friction between the particle and surface is $\frac{3}{4}$.

(a) Find the value of $P$.

(b) Find the magnitude and direction of the contact force.

***Working:*** (a) $\mu = \frac{3}{4}$

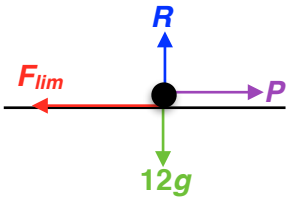
$R(\uparrow)$: $R = 12g$

$F_{lim} = \mu R$: $F_{lim} = \frac{3}{4} \times 12g = 9g$

$F = ma(\rightarrow)$: $P - F_{lim} = 12 \times 3$

$P = 36 + 9g$

$P = 124.2$ N




(b) $R = 12g$ and $F_{lim} = 9g$

$C = \sqrt{R^2 + F_{lim}^2} = \sqrt{(12g)^2 + (9g)^2} = 15g = 147$ N

$\tan^{-1}\frac{12g}{9g} = 53.1^o$

The contact force is of magnitude $147$ N, acting at an angle of $53.1^o$ to the horizontal, opposite to the direction of motion.

Video: Friction

Video: Motion on rough horizontal plane

Motion on rough horizontal plane EQ

Solutions to Starter and E.g.s

**Exercise**

p481 21B Qu 1i, 2-9