Derivative of e^x and $\ln x$

Starter

We have already looked at the rate of change of e^x at AS level.

1. **(Review of AS material)** Copy and complete: the rate of change of e^x is…

   Working: …e^x.

2. **(Review of AS material)** The number of bacteria in a petri dish is given by $N = e^{5t}$. Write down the rate of change of N.

   Working: $5e^{5t} = 5N$

3. Let $y = 6e^{9x}$. Find $\frac{dy}{dx}$.

   Working: $\frac{dy}{dx} = 54e^{9x}$

4. State the derivatives of: (a) e^x (b) Ae^{kx}

   Working: (a) e^x (b) Ake^{kx}

5. Express $\ln x^4$ in the form $k \ln x$.

   Working: $4 \ln x$

6. Simplify: (a) $e^{\ln x}$ (b) $\ln(e^x)$ (c) $e^{7 \ln x}$

   Working: (a) x (b) x (c) x^7

E.g. 1 Find $\frac{dy}{dx}$ given that: (a) $y = 2e^x$ (b) $y = 5e^{7x}$ (c) $y = \frac{9}{e^x}$

Hint: For (c), transform it so that e^x is in the numerator.

Working: (a) $\frac{dy}{dx} = 2e^x$

(b) $\frac{dy}{dx} = 5 \times 7e^{7x} = 35e^{7x}$

(c) Transform to $y = 9e^{-x}$ so $\frac{dy}{dx} = -9e^{-x} = -\frac{9}{e^x}$

E.g. 2 Find the $x-$coordinate of the stationary point for the curve $y = e^{3x} - 6x$ and determine the nature of the point. Leave your answer in exact form.

Working: $\dfrac{dy}{dx} = 3e^{3x} - 6$

A SP occurs when $\dfrac{dy}{dx} = 0$ so $3e^{3x} - 6 = 0$

Solving: $e^{3x} = 2$

$3x = \ln 2$

$x = \dfrac{1}{3}\ln 2$

To determine the nature of the SP: $\dfrac{d^2y}{dx^2} = 9e^{3x}$

Since $\dfrac{d^2y}{dx^2} > 0$ for all values of x the SP must be a minimum

E.g. 3 Find the derivative of $y = \ln kx$

Working: The first time we differentiate $\ln kx$ we will use laws of logs but after that we can just use the result

$y = \ln kx = \ln k + \ln x$

$\ln k$ is just a constant so when we differentiate, it becomes zero

So $\dfrac{dy}{dx} = 0 + \dfrac{1}{x} = \dfrac{1}{x}$

E.g. 4 Find $f'(x)$: (a) $f(x) = 8\ln x$ (b) $f(x) = \dfrac{2}{5}\ln x$

(c) $y = \ln 6x$ (d) $f(x) = \ln\left(\dfrac{9}{x}\right)$

Working: (a) $f'(x) = \dfrac{8}{x}$

(b) $f'(x) = \dfrac{2}{5x}$

(c) $f'(x) = \dfrac{1}{x}$

(b) Using laws of logs: $f(x) = \ln 9 - \ln x$

So $f'(x) = -\dfrac{1}{x}$

E.g. 5 Find equation of the normal to the curve $y = \ln 3x$ at the point where $x = 4$. Give your answer exactly.

Working:

$$\frac{dy}{dx} = \frac{1}{x}$$

When $x = 4$, $\frac{dy}{dx} = \frac{1}{4}$

So gradient of normal is -4

When $x = 4$, $y = \ln 12$

Using $y - y_1 = m(x - x_1)$: $\quad y - \ln 12 = -4(x - 4)$

$y = -4x + 16 + \ln 12$

Video: Derivative of e^x

Video: Derivative of ln x

Solutions to Starter and E.g.s

Exercise

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