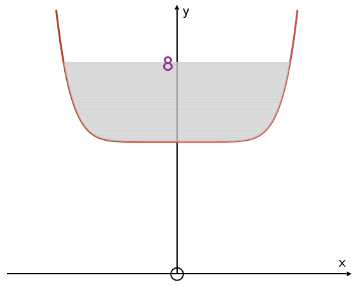
Differentiating Inverse Functions

Starter

1. **(Review of last lesson)**

The curve shown is $y = 3x^8 + 5$. Find the shaded area.



Working: Rearrange $y = 3x^8 + 5$ to make x the subject:

$$x = \sqrt[8]{\frac{y-5}{3}}$$

$$\text{Required area } = 2\int_5^8 \sqrt[8]{\frac{y-5}{3}}\,dy$$

$$= 2\int_5^8 \left(\frac{y-5}{3}\right)^{\frac{1}{8}} dy$$

$$= 2\left[\frac{8}{9}\times 3\left(\frac{y-5}{3}\right)^{\frac{9}{8}}\right]_5^8$$

$$= 2\left[\left(\frac{8}{9}\times 3\left(\frac{8-5}{3}\right)^{\frac{9}{8}}\right) - \left(\frac{8}{9}\times 3\left(\frac{5-5}{3}\right)^{\frac{9}{8}}\right)\right]$$

$$= \frac{48}{9}$$

$$= 5\frac{1}{3}$$

N.B. The integral needs to be multiplied by 2 so that the areas to the left and right of the $y-$axis are counted.
The area could also be found by taking the area between the curve and the $x-$axis away from rectangle

i.e. $$2\int_5^8 \sqrt[8]{\frac{y-5}{3}}\,dy = 1\times 8 - \int_0^1 (3x^8+5)\,dx$$

2. Let $y = \dfrac{5x-3}{2}$. Find:

(a) $\dfrac{dy}{dx}$

(b) the inverse function

(c) the derivative of the inverse function.

Write down what you notice.

Working: (a) $$y = \frac{5x-3}{2} = \frac{5x}{2} - \frac{3}{2}$$

$$\frac{dy}{dx} = \frac{5}{2}$$

(b) To find the inverse: swap x and y over to get $x = \frac{5y-3}{2}$

Rearrange $x = \frac{5y-3}{2}$ to make y the subject: $2x = 5y - 3$

$$2x + 3 = 5y$$
$$y = \frac{2x+3}{5}$$

Inverse function is $\frac{2x+3}{5}$

(c) The derivative of the inverse function is $\frac{2}{5}$.

The derivative of the inverse function is the reciprocal of the derivative of the function.

E.g. 1 Use the derivative of e^x to prove that the derivative of $\ln x$ is $\frac{1}{x}$.

Working: Let $y = \ln x$ then $x = e^y$

$$\frac{dx}{dy} = e^y = e^{\ln x} = x$$

So $\frac{dy}{dx} = \frac{1}{x}$

E.g. 2 The deceleration of a parachute is proportional to the square of its velocity, so that $a = \frac{dv}{dt} = -0.05v^2$. When $t = 0$ the velocity is 12 m/s. Find an expression for the velocity in terms of time.

Hint: $\frac{dt}{dv} = -\frac{1}{0.05v^2}$ so $t = \int \ldots$

Working: $\frac{dt}{dv} = -\frac{1}{0.05v^2} \quad \Rightarrow \quad t = \int -\frac{1}{0.05v^2} dt = \int -20v^{-2} dt$

$$t = 20v^{-1} + c = \frac{20}{v} + c$$

When $t = 0, v = 12$: $\quad 0 = \frac{20}{12} + c \quad \Rightarrow \quad c = -\frac{5}{3}$

$$t = \frac{20}{v} - \frac{5}{3} \quad \Rightarrow \quad 3t + 5 = \frac{60}{v}$$

$$v = \frac{60}{3t+5}$$

Video A: Differentiating inverse functions
Video B: Differentiating inverse functions

Solutions to Starter and E.g.s

Exercise
p215 10E Qu 1-5