Geometric Series

Starter

1. **(Review of last lesson)** Winston invests a sum of money at 6% per annum. How many complete years does it take him to double his money?

Working: Let the sum of money be p.
After n years his investment is worth $p \times 1.06^n$.
To double his money: $p \times 1.06^n = 2p \quad \Rightarrow \quad 1.06^n = 2$
Take* ln *of both sides: $\ln 1.06^n = \ln 2$
3rd law of logs: $n \ln 1.06 = \ln 2$
Rearrange: $n = \dfrac{\ln 2}{\ln 1.06} \qquad n = 11.9$
It takes 12 years to double his investment.

2. **(Review of last lesson)** A geometric sequence has first term 25000 and common ratio 0.8. Which term is the first to be below 1000?

Working: $a = 25000, r = 0.8$
$u_n < 1000$: $25000 \times 0.8^{n-1} < 1000$
$0.8^{n-1} < \dfrac{1}{25}$
Take* ln *of both sides: $\ln 0.8^{n-1} < \ln \dfrac{1}{25}$
3rd law of logs: $(n-1)\ln 0.8 < \ln \dfrac{1}{25}$
$n \ln 0.8 - \ln 0.8 < \ln \dfrac{1}{25}$
$n \ln 0.8 < \ln \dfrac{1}{25} + \ln 0.8$
$n > \dfrac{\ln \frac{1}{25} + \ln 0.8}{\ln 0.8}$

N.B. Since $\ln 0.8 < 0$, when we divide by $\ln 0.8$ the direction of the inequality sign must change.

$n > 15.4$

So the first term below 1000 is the 16th term.

E.g. 1 Find the sum of the first 9 terms of the series 2, 10, 50.

Working: $a = 2, r = \dfrac{10}{2} = \dfrac{50}{10} = 5, n = 9$
Since $r > 1$ use $S_n = \dfrac{a(r^n - 1)}{r - 1}$: $\qquad S_9 = \dfrac{2(5^9 - 1)}{5 - 1} = 976562$

E.g. 2 The sum of the first n terms of a geometric series is 196 605. Given that the common ratio is 2 and the first term is 3. Find n.

Working: $a = 3, r = 2, S_n = 196605$

Since $r > 1$ use $S_n = \dfrac{a(r^n - 1)}{r - 1}$: $\quad \dfrac{3(2^n - 1)}{2 - 1} = 196605$

$2^n = 65536 \Rightarrow \ln 2^n = \ln 65536 \Rightarrow n \ln 2 = \ln 65536$

$n = \dfrac{\ln 65536}{\ln 2} = 16$

E.g. 3 The 3rd term of a geometric series is 6 and the 8th term is 192. Find the sum of the first 15 terms.

Hint: find a and r first.

Working: $u_n = ar^{n-1}$

3rd term is 6: $\quad ar^{3-1} = 6 \Rightarrow ar^2 = 6$

8th term is 192: $\quad ar^{8-1} = 192 \Rightarrow ar^7 = 192$

Divide: $\quad \dfrac{ar^7}{ar^2} = \dfrac{192}{6} \Rightarrow r^5 = 32 \quad \therefore r = 2$

Substituting into $ar^2 = 6$: $\quad a \times 2^2 = 6 \Rightarrow a = \dfrac{6}{4} = 1.5$

Since $r > 1$ use $S_n = \dfrac{a(r^n - 1)}{r - 1}$: $\quad S_{15} = \dfrac{1.5(2^{15} - 1)}{2 - 1} = 49150.5$

The sum of the first 15 terms is 49 150.5.

E.g. 4 Find $\displaystyle\sum_{n=1}^{12} (3 \times 4^n)$

Working: The 1st term is when $n = 1$: $\quad 3 \times 4^1 = 12$

The terms are 12, 48, 192, 768.

This is a geometric progression with $a = 12$ and $r = 4$

Since $r > 1$ use $S_n = \dfrac{a(r^n - 1)}{r - 1}$: $\quad S_{12} = \dfrac{12(4^{12} - 1)}{4 - 1} = 67108860$

Video: Sum of geometric series

Solutions to Starter and E.g.s

Exercise

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