

Hypothesis tests for a mean

Starter

1. (a) If $X_1, X_2, X_3, \dots, X_n$ is a random sample from $N(\mu, 1)$, state the distribution of the sample mean \bar{X} .
- (b) Find the least sample size required to ensure that the probability that \bar{X} is within 0.1 of μ is greater than 0.95.

Working:

(a) $X \sim N(\mu, 1) \Rightarrow \bar{X}_n \sim N\left(\mu, \frac{1}{n}\right)$

(b) $P(|\bar{X} - \mu| < 0.1) = 0.95 \Rightarrow P(\mu - 0.1 < \bar{X} < \mu + 0.1) = 0.95$
 $P(\bar{X} < \mu + 0.1) = 0.95 + \frac{0.05}{2} = 0.975$
 Let z be the z -value corresponding to $\mu + 0.1$.
 $P(Z < z) = 0.975 \Rightarrow z = 1.96$
 Substitute into $z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}}$: $1.96 = \frac{\mu + 0.1 - \mu}{\sqrt{\frac{1}{n}}}$
 $1.96 = 0.1 \times \sqrt{n}$
 $n \approx 384.1$

The sample size required is 385.

E.g. 1 Use the “ p -value” method to test these hypotheses at the stated levels:

- (a) $H_0 : \mu = 5, H_1 : \mu \neq 5, n = 25, \bar{x} = 6.1, \sigma = 3.0$ at the 5 % level
- (a) $H_0 : \mu = 100, H_1 : \mu < 100, n = 36, \bar{x} = 98.5, \sigma = 5$ at the 5 % level
- (a) $H_0 : \mu = 15, H_1 : \mu > 15, n = 40, \bar{x} = 16.5, \sigma = 3.5$ at the 1 % level

Working:

(a) $X \sim N(5, 3.0^2) \Rightarrow \bar{X}_{25} \sim N\left(5, \frac{3.0^2}{25}\right)$
 Since $\bar{x} = 6.1$ is **greater than** $\mu = 5$, calculate $P(\bar{X} > 6.1)$.
 $p = P(\bar{X} > 6.1) = 0.0334 \equiv 3.34\%$
 Since it is a two-tailed test, $\alpha = \frac{5\%}{2} = 2.5\%$
 Since $p = 3.34\% > 2.5\%$, we **do not reject** H_0 at the 5 % level.

(b) $X \sim N(100, 5^2) \Rightarrow \bar{X}_{36} \sim N\left(100, \frac{5^2}{36}\right)$
 $p = P(\bar{X} < 98.5) = 0.0359 \equiv 3.59\%$
 Since $p = 3.59\% < 5\%$, we **reject** H_0 at the 5 % level.

(c) $X \sim N(15, 3.5^2) \Rightarrow \bar{X}_{40} \sim N\left(15, \frac{3.5^2}{40}\right)$
 $P(\bar{X} > 16.5) = 0.00336 \equiv 0.336\%$
 Since $p = 0.336\% < 1\%$, we **reject** H_0 at the 1 % level.

E.g. 2 Use the “critical region” method to test these hypotheses at the stated levels:

- (a) $H_0 : \mu = 120, H_1 : \mu < 120, n = 30, \bar{x} = 119, \sigma = 2.0$ at the 5 % level.
 (b) $H_0 : \mu = 12.5, H_1 : \mu > 12.5, n = 25, \bar{x} = 12.9, \sigma = 1.5$ at the 1 % level.
 (c) $H_0 : \mu = 0, H_1 : \mu \neq 0, n = 45, \bar{x} = 0.9, \sigma = 3.0$ at the 5 % level.

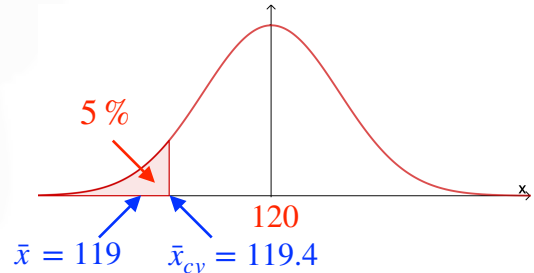
Draw a separate diagram for each question.

Working: (a) $X \sim N(120, 2.0^2) \Rightarrow \bar{X}_{30} \sim N\left(120, \frac{2.0^2}{30}\right)$

Let \bar{x}_{cv} be the critical value such that $P(\bar{X} < \bar{x}_{cv}) = 0.05$

$\Rightarrow \bar{x}_{cv} = 119.4$

Since $\bar{x} = 119 < 119.4 = \bar{x}_{cv}$, the **test statistic lies in the critical region** so we **reject H_0** .

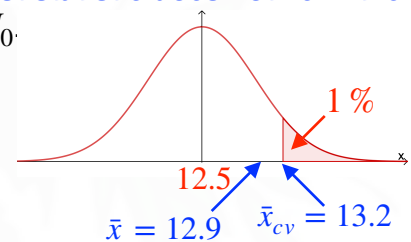


(b) $X \sim N(12.5, 1.5^2) \Rightarrow \bar{X}_{25} \sim N\left(12.5, \frac{1.5^2}{25}\right)$

Let \bar{x}_{cv} be the critical value such that $P(\bar{X} > \bar{x}_{cv}) = 0.05$.

$P(\bar{X} < \bar{x}_{cv}) = 1 - 0.05 = 0.95 \Rightarrow \bar{x}_{cv} = 13.2$

Since $\bar{x} = 12.9 < 13.2 = \bar{x}_{cv}$, the **test statistic does not lie in the critical region** so we **do not reject H_0** .

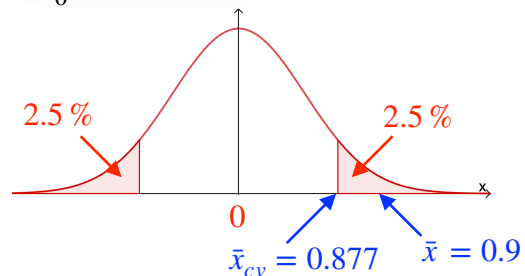


(c) $X \sim N(0, 3.0^2) \Rightarrow \bar{X}_{45} \sim N\left(0, \frac{3.0^2}{45}\right)$

Let \bar{x}_{cv} be the critical value such that $P(\bar{X} > \bar{x}_{cv}) = 0.025$.

$P(\bar{X} < \bar{x}_{cv}) = 1 - 0.025 = 0.975 \Rightarrow \bar{x}_{cv} = 0.877$

Since $\bar{x} = 0.9 > 0.877 = \bar{x}_{cv}$, the **test statistic lies in the critical region** so we **reject H_0** .



N.B. When you do worded questions make sure that you write H_0 and H_1 clearly.

E.g. 3 The IQ scores of a population are normal distributed with a mean of 100 and standard deviation of 15. A psychologist wishes to test the theory that eating chocolate before sitting an IQ test improves your score. A random sample of 81 people are selected and they are each given a 50 g bar of chocolate to eat before taking a standard IQ test. Their mean score on the test was 103.1. Test the psychologist's theory at the 2% level.

Working: $X \sim N(100, 15^2) \Rightarrow \bar{X}_{81} \sim N\left(100, \frac{15^2}{81}\right)$

The test statistic is $\bar{x} = 102.5$.

$$H_0 : \mu = 100$$

$$H_1 : \mu > 100$$

$$p = P(\bar{X} > 103.1) = 0.0314 \equiv 3.14 \%$$

Since $p = 3.14 \% > 2 \%$, we **do not reject** H_0 at the 2% level.

There is evidence to suggest that eating chocolate before an IQ test does not improve performance.

E.g. 4 The mean weight of men in a particular country is found to have a standard deviation of 9.1 kg. The sample mean based on a random sample of 120 men was found to be 75.3 kg. A medical journal claims that the mean weight of men in this country is 77.1 kg. Test the validity of this statement at the 5% level.

Working: $X \sim N(77.1, 9.1^2) \Rightarrow \bar{X}_{120} \sim N\left(77.1, \frac{9.1^2}{120}\right)$

The test statistic is $\bar{x} = 75.3$.

$$H_0 : \mu = 77.1$$

$$H_1 : \mu < 77.1$$

Let \bar{x}_{cv} be the critical value such that $P(\bar{X} < \bar{x}_{cv}) = 0.05$. $\Rightarrow \bar{x}_{cv} = 75.7$

Since $\bar{x} = 75.3 < 75.7 = \bar{x}_{cv}$, the **test statistic lies in the critical region** so we **reject** H_0 .

There is no evidence to suggest that the mean weight of men in the country is 77.1 kg as claimed by the medical journal.

Video: [An introduction to hypothesis testing](#)

Video: [Mean of Normal distribution hypothesis testing](#)

Video: [Sample means: hypothesis testing example](#)

[Solutions to Starter and E.g.s](#)

Exercise

p405 18B Qu 1i, 2i, 3i, 4-8, (9 red)