Hypothesis tests for correlation coefficients

Starter

1. A trading standards officer finds that the mean lifetime of a random sample of 50 long lasting light bulbs manufactured by En-Lightened Ltd is 1700 hours rather than the 1800 hours claimed by the company. The light bulbs are known to have a standard deviation of 125 hours. The officer carries out a hypothesis test at the $5\,\%$ level believing that light bulbs last shorter than the company claims. What conclusion does the officer reach?

Working: $X \sim \text{N}(1800,\, 125^2) \quad \Rightarrow \quad \bar{X}_{150} \sim \text{N}\left(1800,\, \dfrac{125^2}{50}\right)$

The test statistic is $\bar{x} = 1770$.
$H_0 : \mu = 1800$
$H_1 : \mu < 1800$
$p = P(\bar{X} < 1770) = 0.0448 \equiv 4.48\,\%$
Two-tailed test so halve the significance level.
Since $p = 4.48\,\% < 5\,\%$, we ***reject*** H_0 at the $5\,\%$ level.
There is evidence to suggest that the light bulbs do not last an average of 1800 hours as claimed by the company.

E.g. 1 Carry out a one-tailed hypothesis test for these PMCC values at the given level of significance, where n is the sample size:
(a) $r = 0.465,\, n = 18$ at the $5\,\%$
(b) $r = 0.31,\, n = 45$ at the $1\,\%$
Write down the null and alternative hypotheses each time.

Working:

(a) $H_0 : \rho = 0$
$H_1 : \rho > 0$
From tables, for a ***one-tailed test*** at the $5\,\%$ level where $n = 18$, the critical value is 0.4000.
Since $|r| = 0.465 > 0.4000$, reject H_0
There is no evidence to suggest that there is positive correlation between the variables.

(b) $H_0 : \rho = 0$
$H_1 : \rho > 0$
From tables, for a ***one-tailed test*** at the $1\,\%$ level where $n = 45$, the critical value is 0.3457.
Since $|r| = 0.31 < 0.3457$, do not reject H_0
There is no evidence to suggest that there is positive correlation between the variables.

E.g. 2 Carry out a one-tailed hypothesis test for these PMCC values at the given level of significance, where n is the number of paired bivariate data:
(a) $r = -0.37$, $n = 23$ at the 2.5%
(b) $r = -0.89$, $n = 6$ at the 5%
Write down the null and alternative hypotheses each time.

Working: (a)
$H_0 : \rho = 0$
$H_1 : \rho < 0$
From tables, for a ***one-tailed test***, at the 2.5% level where $n = 23$, the critical value is 0.4132.
Since $|r| = 0.37 < 0.4132$, do not reject H_0
There is no evidence to suggest that there is negative correlation between the variables.

(b)
$H_0 : \rho = 0$
$H_1 : \rho < 0$
From tables, for a ***one-tailed test*** at the 5% level where $n = 6$, the critical value is 0.8822.
Since $|r| = 0.89 > 0.8822$, do not reject H_0.
There is evidence to suggest that there is negative correlation between the variables.

E.g. 3 Carry out a two-tailed hypothesis test for these PMCC values at the given level of significance, where n is the sample size:
(a) $r = 0.751$, $n = 8$ at the 1%
(b) $r = -0.89$, $n = 11$ at the 5%
Write down the null and alternative hypotheses each time.

Working: (a)
$H_0 : \rho = 0$
$H_1 : \rho \neq 0$
From tables, for a ***two-tailed test*** at the 1% level where $n = 8$, the critical value is 0.8343.
Since $|r| = 0.751 < 0.8343$, do not reject H_0
There is no evidence to suggest that there is correlation between the variables.

(b)
$H_0 : \rho = 0$
$H_1 : \rho \neq 0$
From tables, for a ***two-tailed test*** at the 5% level where $n = 11$, the critical value is 0.6021.
Since $|r| = 0.89 > 0.8822$, do not reject H_0.
There is evidence to suggest that there is correlation between the variables.

E.g. 4 A farming cooperative plotted mass of wheat crop yield against rainfall for 9 different areas of the country. The correlation coefficient for the data was 0.659.

(a) Test at the $5\,\%$ level whether rainfall improves wheat crop. State the null and alternative hypotheses clearly.

(b) Give a reason why the a hypothesis test may not be suitable for the data value.

Working: (a) $H_0 : \rho = 0$

$H_1 : \rho > 0$

From tables, for a ***one-tailed test*** at the $5\,\%$ level where $n = 9$, the critical value is 0.5822.

Since $|r| = 0.659 > 0.5822$, reject H_0

There is evidence to suggest there is a positive correlation between rainfall and wheat crop size.

(b) Rainfall and wheat crop yield may not be linearly related. For example, too much rain may lead to less of a yield.

Critical values of Pearson's product-moment correlation coefficient

1-tail test	5%	2.5%	1%	0.5%
2-tail test	10%	5%	2%	1%
n				
1	-	-	-	-
2	-	-	-	-
3	0.9877	0.9969	0.9995	0.9999
4	0.9000	0.9500	0.9800	0.9900
5	0.8054	0.8783	0.9343	0.9587
6	0.7293	0.8114	0.8822	0.9172
7	0.6694	0.7545	0.8329	0.8745
8	0.6215	0.7067	0.7887	0.8343
9	0.5822	0.6664	0.7498	0.7977
10	0.5494	0.6319	0.7155	0.7646
11	0.5214	0.6021	0.6851	0.7348
12	0.4973	0.5760	0.6581	0.7079
13	0.4762	0.5529	0.6339	0.6835
14	0.4575	0.5324	0.6120	0.6614
15	0.4409	0.5140	0.5923	0.6411
16	0.4259	0.4973	0.5742	0.6226
17	0.4124	0.4821	0.5577	0.6055
18	0.4000	0.4683	0.5425	0.5897
19	0.3887	0.4555	0.5285	0.5751
20	0.3783	0.4438	0.5155	0.5614
21	0.3687	0.4329	0.5034	0.5487
22	0.3598	0.4227	0.4921	0.5368
23	0.3515	0.4132	0.4815	0.5256
24	0.3438	0.4044	0.4716	0.5151
25	0.3365	0.3961	0.4622	0.5052
26	0.3297	0.3882	0.4534	0.4958
27	0.3233	0.3809	0.4451	0.4869
28	0.3172	0.3739	0.4372	0.4785
29	0.3115	0.3673	0.4297	0.4705
30	0.3061	0.3610	0.4226	0.4629

Video: Hypothesis testing for zero correlation
Video: Correlation hypothesis testing

Solutions to Starter and E.g.s

Exercise
p410 18C Qu 1i, 2i, 3-7, (8-9 red)