Integrals involving Brackets

Starter

1. (a) Differentiate $y = (3x+5)^8$.

   (b) Using your answer to (a) integrate the function $\int (3x+5)^7 dx$.

Working: (a) $\frac{dy}{dx} = 24(3x+5)^7$ (b) $\frac{1}{24}(3x+5)^8 + c$

E.g. 1 Find: (a) $\int (3x+5)^7 dx$ (b) $\int k(ax+b)^n dx$

Working: (a) Let $u = 3x+5 \Rightarrow \frac{du}{dx} = 3 \Rightarrow dx = \frac{du}{3}$

$$\int (3x+5)^7 dx = \int \frac{1}{3} u^7 du$$ ***replace*** $3x+5$ ***and*** dx

$$= \frac{1}{24} u^8 + c$$ ***integrate with respect to*** u

$$= \frac{1}{24}(3x+5)^8 + c$$ ***replace*** u ***by*** $3x+5$

(b) Let $u = ax+b \Rightarrow \frac{du}{dx} = a \Rightarrow dx = \frac{du}{a}$

$$\int k(ax+b)^n dx = \int \frac{1}{a} u^n du$$ ***replace*** $ax+b$ ***and*** dx

$$= \frac{1}{a(n+1)} u^{n+1} + c$$ ***integrate wrt*** u

$$= \frac{1}{a(n+1)}(ax+b)^{n+1} + c$$ ***replace*** u ***by*** $ax+b$

E.g. 2 Does $\int (x^2+3)^6 dx = \frac{1}{14x} \times (x^2+3)^7 + c$?

Working: No, we would need to use the quotient rule to differentiate the function.

E.g. 3 Find: (a) $\int 3(1-5x)^6 dx$ (b) $\int \frac{1}{(2x-9)^6} dx$ (c) $\int \sqrt{7x+5}\, dx$

Working: (a) $\int 3(1-5x)^6 dx = = \frac{3}{(-5)\times 7}(1-5x)^7 + c = -\frac{3}{35}(1-5x)^7 + c$

(b) $$\int \frac{1}{(2x-9)^6} dx = \int (2x-9)^{-6} dx$$

$$= -\frac{1}{2\times(-5)}(2x-9)^{-5} + c$$

$$= -\frac{1}{10(2x-9)^5} + c$$

(c) $$\int \sqrt{7x+5}\,dx = \int (7x+5)^{\frac{1}{2}}dx$$
$$= \frac{1}{7\times\frac{3}{2}}(7x+5)^{\frac{3}{2}} + c$$
$$= \frac{2}{21}\sqrt{(7x+5)^3} + c$$

E.g. 4 Find: (a) $\displaystyle\int_1^{1.5} (3-2x)^3 dx$ (b) $\displaystyle\int_0^1 \frac{1}{(6x+1)^3}dx$

Working: (a) $$\int_1^{1.5} (3-2x)^3 dx = \left[\frac{1}{(-2)\times 4}(3-2x)^4 dx\right]_1^{1.5}$$
$$= \frac{1}{8}\left[(3-2x)^4\right]_{1.5}^{1}$$

N.B. ***Changing the limits around, changes the sign of the integral.***

$$= \frac{1}{8}\left[(3-2\times 1)^4 - (3-2\times 1.5)^4\right]$$
$$= \frac{1}{8}$$

(b) $$\int_0^1 \frac{1}{(6x+1)^3}dx = \int_0^1 (6x+1)^{-3}dx$$
$$= \left[\frac{1}{6\times(-2)}(6x+1)^{-2}\right]_0^1$$
$$= \frac{1}{12}\left[\frac{1}{(6x+1)^2}\right]_1^0$$
$$= \frac{1}{12}\left[\frac{1}{1} - \frac{1}{(6+1)^2}\right]$$
$$= \frac{1}{12}\times\frac{48}{49}$$
$$= \frac{4}{49}$$

E.g. 5 The curve $y = f(x)$ goes through the point $\left(1, \frac{3}{35}\right)$ and $f'(x) = (8-7x)^4$. Find $f(x)$.

Working: $$f(x) = \int (8-7x)^4 dx = \frac{1}{(-7)\times 5}(8-7x)^5 + c = -\frac{1}{35}(8-7x)^5 + c$$

$\left(1, \frac{3}{35}\right)$: $\quad \frac{3}{35} = -\frac{1}{35}(8-7\times 1)^5 + c \quad \Rightarrow \quad \frac{3}{35} = -\frac{1}{35} + c$

So $c = \frac{4}{35}$, $\therefore f(x) = -\frac{1}{35}(8-7x)^5 + \frac{4}{35}$

Video: Integration by inspection (brackets)

Integration by inspection (brackets) EQ

Solutions to Starter and E.g.s

Exercise
p222 11A Qu 1