Integrals involving Trigonometry

Notes

1. **(Review of last lesson)** Find: (a) $\int 8e^{2-7x}dx$ (b) $\int \frac{1}{7x+2}dx$

Working: (a) $-\frac{8}{7}e^{2-7x}+c$

(b) $\frac{1}{7}\ln(7x+2)+c$

2. By using "Let $u = \ldots$" find the following:

(a) $\int \sin(2x+1)dx$ (b) $\int \sec^2(6x-5)dx$

(c) $\int \cos(ax+b)dx$ (d) $\int \sin(x^2-7)dx$

Working: (a) Let $u = 2x+1 \quad \Rightarrow \quad \frac{du}{dx} = 2 \quad \Rightarrow \quad \frac{du}{2} = dx$

$$\int \sin(2x+1)dx = \int \frac{1}{2}\sin u\,du \qquad \textit{replace } 2x+1 \textit{ and } dx$$
$$= -\frac{1}{2}\cos u + c \qquad \textit{integrate with respect to } u$$
$$= -\frac{1}{2}\cos(2x+1) + c \qquad \textit{replace } u \textit{ by } 2x+1$$

(b) Let $u = 6x-5 \quad \Rightarrow \quad \frac{du}{dx} = 6 \quad \Rightarrow \quad \frac{du}{6} = dx$

$$\int \sec^2(6x+5)dx = \int \frac{1}{6}\sec^2 u\,du \qquad \textit{replace } 6x+5 \textit{ and } dx$$
$$= \frac{1}{6}\tan u + c \qquad \textit{integrate with respect to } u$$
$$= \frac{1}{6}\tan(6x-5) + c \qquad \textit{replace } u \textit{ by } 6x+5$$

(c) Let $u = ax+b \quad \Rightarrow \quad \frac{du}{dx} = a \quad \Rightarrow \quad \frac{du}{a} = dx$

$$\int \cos(ax+b)dx = \int \frac{1}{a}\cos u\,du \qquad \textit{replace } 2x+1 \textit{ and } dx$$
$$= \frac{1}{a}\sin u + c \qquad \textit{integrate with respect to } u$$
$$= \frac{1}{a}\sin(ax+b) + c \qquad \textit{replace } u \textit{ by } 2x+1$$

(d) Not possible since the function in brackets is not linear.

E.g. 1 Find: (a) $\int \sin(9x-4)dx$ (b) $\int \sec^2(3x-11)dx$

Working: (a) $-\frac{1}{9}\cos(9x-4)+c$

(b) $\frac{1}{3}\tan^2(3x-11)+c$

E.g. 2 Find: (a) $\int_1^{1.5} \cos(2x-1)dx$ (to 3 s.f.) (b) $\int_0^{\frac{\pi}{6}} (\cos 3x+\sin 2x)dx$

Working: (a) $$\int_1^{1.5} \cos(2x-1)dx = \frac{1}{2}\Big[\sin(2x-1)\Big]_1^{1.5}$$
$$= \frac{1}{2}\Big[\sin(2\times 1.5-1)-\sin(2\times 1-1)\Big]$$
$$= \frac{1}{2}(\sin 2^c - \sin 1^c) \approx 0.0339$$

Remember 2^c means 2 radians

(b) $$\int (\cos 3x+\sin 2x)dx = \Big[\frac{1}{3}\sin 3x - \frac{1}{2}\cos 2x\Big]_0^{\frac{\pi}{6}}$$
$$= \left(\frac{1}{3}\sin\frac{\pi}{2} - \frac{1}{2}\cos\frac{\pi}{3}\right) - \left(\frac{1}{3}\sin 0 - \frac{1}{2}\cos 0\right)$$
$$= \left(\frac{1}{3} - \frac{1}{2}\times\frac{1}{2}\right) - \left(0-\frac{1}{2}\right)$$
$$= \frac{1}{3} - \frac{1}{4} + \frac{1}{2}$$
$$= \frac{7}{12}$$

Video: Integrals of sin/cos/tan with brackets

Solutions to Starter and E.g.s

Exercise
p222 11A Qu 4a-e, 5-8