Integrals of Trigonometric Functions

Starter

1. **(Review of last lesson)**

A curve has gradient $\frac{4}{3x}$ and passes through the point (2, 7). Susan says the equation of the curve could be either $y = \frac{4}{3}\ln x + 6.08$ or $y = \frac{4}{3}\ln 3x + 4.61$. Tom said she is wrong because the equation must be unique. Who is correct and why?

Working: $\frac{dy}{dx} = \frac{4}{3x} \quad \Rightarrow \quad y = \int \frac{4}{3x} dx = \frac{4}{3}\ln x + c$

Substitute (2, 7): $7 = \frac{4}{3}\ln 2 + c \qquad \therefore c \approx 6.08$

So $y = \frac{4}{3}\ln x + 6.08$

Now $y = \frac{4}{3}\ln 3x + 4.61 = \frac{4}{3}\ln x + \frac{4}{3}\ln 3 + 4.61 = \frac{4}{3}\ln x + 6.08$

Susan is correct — they are the same equation since $\frac{4}{3}\ln 3 + 4.61 = 6.08$

2. Given that $\frac{d(\sin x)}{dx} = \cos x$, $\frac{d(\cos x)}{dx} = -\sin x$ and $\frac{d(\tan x)}{dx} = \sec^2 x$, find:

(a) $\int k \sin x \, dx$ (b) $\int k \cos x \, dx$ (c) $\int k \sec^2 x \, dx$.

Working:

(a) $\int k \sin x \, dx = k \int \sin x \, dx = -k \cos x + c$

(b) $\int k \cos x \, dx = k \int \cos x \, dx = k \sin x + c$

(c) $\int k \sec^2 x \, dx = k \int \sec^2 x \, dx = k \tan x + c$

E.g. 1 Find: (a) $\int \frac{1}{7}\cos x \, dx$ (b) $\int 6 \sin x - 7 \sec^2 x \, dx$

Working:

(a) $\int \frac{1}{7}\cos x \, dx = \frac{1}{7}\sin x + c$

(b) $\int 6 \sin x - 7 \sec^2 x \, dx = -6 \cos x - 7 \tan x + c$

E.g. 2 Find $\int_0^{\frac{\pi}{6}} 5\cos x\,dx$.

Working:

$$\int_0^{\frac{\pi}{6}} 5\cos x\,dx = \left[5\sin x\right]_0^{\frac{\pi}{6}}$$
$$= 5\sin\frac{\pi}{6} - 5\sin 0$$
$$= 5 \times \frac{1}{2 - 0}$$
$$= \frac{5}{2}$$

E.g. 3 Find k such that $\int_k^{\frac{\pi}{3}} 7\sin x\,dx = \frac{1}{3}$. Give your answer to 3 s.f.

Working:

$$\int_k^{\frac{\pi}{3}} 7\sin x\,dx = \left[-7\cos x\right]_k^{\frac{\pi}{3}}$$
$$= \left[7\cos x\right]_{\frac{\pi}{3}}^{k}$$ (swapping the limits, changes the sign)
$$= 7\cos k - 7\cos\frac{\pi}{3}$$

So $7\left(\cos k - \cos\frac{\pi}{3}\right) = \frac{1}{3}$

Rearranging gives $k \approx 0.991^c$ (have your calculator in radians)

Video: Integrating sin/cos/tan

Solutions to Starter and E.g.s

Exercise
p193 9B Qu 1idef, 3ifg, 5, 6, 9, 12 (not Qu 10)

Qu 12(a) You need to use your calculator to solve $e^x - 5\sin x = 0$.

Video: Solving equations with Classwiz

Make sure your calculator is in radians.

Enter $e^x - 5\sin x = 0$ on your Classwiz using

Alpha >>) to enter "x"
ALPHA >> CALC to enter "="

Then press SHIFT >> CALC to SOLVE

For some reason your calculator throws up an old value for x that is in its memory. Press "=" to get the actual value you want which is 0.263265.