Integrating $\sin^2 x$ and $\cos^2 x$

Starter

1. **(Review of A2 material)** Integrate: (a) $\int \sin 6x\,dx$ (b) $\int 7\cos 3x\,dx$

Working: (a) $-\frac{1}{6}\cos 6x + c$ (b) $\frac{7}{3}\sin 3x + c$

2. **(Review of A2 material)** State the double angle identities for $\cos 2x$:
(a) involving $\cos x$ only and (b) involving $\sin x$ only.

Working: (a) $\cos 2x \equiv 2\cos^2 x - 1$ (b) $\cos 2x \equiv 1 - 2\sin^2 x$

3. **(Review of A2 material)**
State the identity (a) for $\cos 4x$ involving $\sin 2x$ (b) $\cos 10x$ involving $\cos 5x$

Working: (a) $\cos 4x \equiv 1 - 2\sin^2 2x$ (b) $\cos 10x \equiv 2\cos^2 5x - 1$

4. Express, in terms of cosine and using double angle formulae, (a) $\cos^2 3x$ (b) $\sin^2 7x$

Working: (a) $\cos^2 3x = \frac{1}{2}(1 + \cos 6x)$

(b) $\sin^2 7x = \frac{1}{2}(1 - \cos 14x)$

5. Hence find: (a) $\int \cos^2 3x\,dx$ (b) $\int \sin^2 7x\,dx$

Working: (a)
$$\begin{aligned}\int \cos^2 3x\,dx &= \int \frac{1}{2}(1 + \cos 6x)dx \\ &= \int \left(\frac{1}{2} + \frac{1}{2}\cos 6x\right)dx \\ &= \frac{1}{2}x + \frac{1}{12}\sin 6x + c\end{aligned}$$

(b)
$$\begin{aligned}\int \sin^2 7x\,dx &= \int \frac{1}{2}(1 - \cos 14x)dx \\ &= \int \left(\frac{1}{2} - \frac{1}{2}\cos 14x\right)dx \\ &= \frac{1}{2}x - \frac{1}{28}\sin 14x + c\end{aligned}$$

E.g. 1 Find $\int \cos^2 5x\,dx.$

Working:

$$\cos 2x = 2\cos^2 x - 1 \quad \Rightarrow \quad \cos 10x = 2\cos^2 5x - 1$$

$$\therefore \cos^2 5x = \frac{1}{2}(1 + \cos 10x) = \frac{1}{2} + \frac{1}{2}\cos 10x$$

$$\int \cos^2 5x\,dx = \int \frac{1}{2}(1 + \cos 10x)\,dx$$

$$= \int \left(\frac{1}{2} + \frac{1}{2}\cos 10x\right)dx$$

$$= \frac{1}{2}x + \frac{1}{20}\sin 10x + c$$

Video: Integration involving sin^2/cos^2
Video: Integration using trigonometric identities

Integrals involving trigonometric functions EQ

Solutions to Starter and E.g.s

Exercise
p238 11F Qu 3a(i&ii), 4a(i&ii), 6