

Integrating $\sin x \cos x$

Starter

1. **(Review of last lesson)** Find: (a) $\int 6 \sin^2 x dx$ (b) $\int 4 + \cot^2 3x dx$.

Working: (a) $\cos 2x = 1 - 2 \sin^2 x \Rightarrow 3 \cos 2x = 3 - 6 \sin^2 x$
 $\therefore 6 \sin^2 x = 3 - 3 \cos 2x$
 $\int 6 \sin^2 x dx = \int 3 - 3 \cos 2x dx$
 $= 3x - \frac{3}{2} \sin 2x + c$

(b) $1 + \cot^2 3x \equiv \operatorname{cosec}^2 3x \Rightarrow 4 + \cot^2 3x \equiv 3 + \operatorname{cosec}^2 3x$
 $\int 4 + \cot^2 3x dx = \int (3 + \operatorname{cosec}^2 3x) dx = 3x - \frac{1}{3} \cot 3x + c$

2. **(Review of A2 material)** State the double angle identity for $\sin 2A$.

Working: $\sin 2A \equiv 2 \sin A \cos A$

3. Express $8 \sin 6x \cos 6x$ in terms of sine only. Hence find $\int 8 \sin 6x \cos 6x dx$.

Working: $8 \sin 6x \cos 6x \equiv 4 \times (2 \sin 6x \cos 6x) \equiv 4 \sin 12x$
 $\int 8 \sin 6x \cos 6x dx = \int 4 \sin 12x dx = -\frac{1}{3} \cos 12x + c$

- E.g. 1** Find: (a) $\int \sin 2x \cos 2x dx$ (b) $\int (\sin x + 2)^2 dx$
(c) $\int 3 \tan^2 4x dx$ (d) $\int 6 \sin 5x \cos 5x dx$

Working: (a) $2 \sin x \cos x \equiv \sin 2x \Rightarrow \sin 2x \cos 2x \equiv \frac{1}{2} \sin 4x$
 $\int \sin 2x \cos 2x dx = \int \frac{1}{2} \sin 4x dx = -\frac{1}{8} \cos 4x + c$

(b) $\int (\sin x + 2)^2 dx = \int (\sin^2 x + 4 \sin x + 4) dx$
 $\cos 2x = 1 - 2 \sin^2 x \Rightarrow \sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$
 $\int (\sin x + 2)^2 dx = \int \left(\frac{1}{2} - \frac{1}{2} \cos 2x + 4 \sin x + 4 \right) dx$
 $= \int \left(\frac{9}{2} - \frac{1}{2} \cos 2x + 4 \sin x \right) dx$
 $= \frac{9}{2} x - \frac{1}{4} \sin 2x - 4 \cos x + c$

$$(c) \quad 1 + \tan^2 4x \equiv \sec^2 4x \quad \Rightarrow \quad 3 \tan^2 4x \equiv 3 \sec^2 4x - 3$$
$$\int 3 \tan^2 4x dx = \int (3 \sec^2 4x - 3) dx = \frac{3}{4} \tan 4x - 3x + c$$

$$(d) \quad 2 \sin x \cos x \equiv \sin 2x \quad \Rightarrow \quad 2 \sin 5x \cos 5x \equiv \sin 10x$$
$$\therefore 6 \sin 5x \cos 5x \equiv 3 \sin 10x$$
$$\int 6 \sin 5x \cos 5x dx = \int 3 \sin 10x dx = -\frac{3}{10} \cos 10x + c$$

Video: [Integration using trigonometric identities](#)

Integrals involving trigonometric functions EQ

[Solutions to Starter and E.g.s](#)

Exercise

No questions in textbook