Integrating $\tan^2 x$ and $\cot^2 x$

Starter

1. (Review of A2 material) Integrate: (a) $\int \sec^2 x dx$ (b) $\int cosec^2 x dx$

Working: (a) $\tan x + c$ (b) $-\cot^2 x + c$

2. State the identity involving $\tan^2 x$ and $\sec^2 x$. Hence find $\int \tan^2 x dx$.

Hint: Start from $\cos^2 x + \sin^2 x \equiv 1$ and divide each term by $\cos^2 x$.

Working: From $\cos^2 x + \sin^2 x \equiv 1$, by dividing each term by $\cos^2 x$ we get:

$1 + \tan^2 x \equiv \sec^2 x \quad \Rightarrow \quad \tan^2 x \equiv \sec^2 x - 1$

$\therefore \int \tan^2 x dx = \int (\sec^2 x - 1) dx = \tan x - x + c$

3. State the identity for $\tan^2 3x$ involving sec. Hence find $\int \tan^2 3x dx$.

Hint: Write $\tan^2 3x = \ldots$

Working: $\tan^2 3x \equiv \sec^2 3x - 1$

$\int \tan^2 3x dx = \int (\sec^2 3x - 1) dx = \frac{1}{3} \tan 3x - x + c$

4. State the identity involving $\cot^2 x$ and $cosec^2 x$. Hence find $\int \cot^2 x dx$.

Hint: Start from $\cos^2 x + \sin^2 x \equiv 1$ and divide each term by $\sin^2 x$.

Working: From $\cos^2 x + \sin^2 x \equiv 1$, by dividing each term by $\sin^2 x$ we get:

$1 + \cot^2 x \equiv cosec^2 x \quad \Rightarrow \quad \cot^2 x \equiv cosec^2 x - 1$

$\int \cot^2 x dx = \int (cosec^2 x - 1) dx = -\cot x - x + c$

E.g. 1 Find $\int \tan^2 3x dx$.

Working: $1 + \tan^2 3x \equiv \sec^2 3x \quad \Rightarrow \quad \tan^2 3x \equiv \sec^2 3x - 1$

$\int \tan^2 3x dx = \int (\sec^2 3x - 1) dx = \frac{1}{3} \tan 3x - x + c$

E.g. 2 Find $\int \cot^2 7x\,dx$

Working: $1 + \cot^2 7x \equiv cosec^2 7x \quad \Rightarrow \quad \cot^2 7x \equiv cosec^2 7x - 1$

$$\int \cot^2 7x\,dx = \int (cosec^2 7x - 1)\,dx = -\frac{1}{7}\cot 7x - x + c$$

Video: Integration using trigonomeric identities

Solutions to Starter and E.g.s

Exercise
p238 11F Qu 3b(i&ii), 4b(i&ii), 8