Integration of e^x and $\frac{1}{x}$

Starter

1. Given that $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\ln x)}{dx} = \frac{1}{x}$, find:

(a) $\int e^x dx$ (b) $\int \frac{1}{x} dx$ (c) $\int Ae^{kx} dx$ (d) $\int \frac{P}{Qx} dx$

Working: (a) $e^x + c$ (b) $\ln|x| + c$

(c) $\frac{A}{k}e^{kx} + c$ (d) $\frac{P}{Q}\ln|x| + c$

E.g. 1 Find: (a) $\int 8e^{3x} dx$ (b) $\int \frac{1}{9}e^{4x} dx$ (c) $\int \frac{1}{8x} dx$ (d) $\int \frac{3}{7x} dx$

Working: (a) $\frac{8}{3}e^{3x} + c$

(b) $\frac{1}{36}e^{4x} + c$

(c) $\int \frac{1}{8x} dx = \frac{1}{8}\int \frac{1}{x} dx = \frac{1}{8}\ln|x| + c$

(d) $\int \frac{3}{7x} dx = \frac{3}{7}\int \frac{1}{x} dx = \frac{3}{7}\ln|x| + c$

E.g. 2 Find the equation of the curve that has derivative $5e^{4x}$ given that the curve passes through the point (2, 6e⁸). Give your answer exactly.

Working: The derivative is $5e^{4x}$ so $\frac{dy}{dx} = 5e^{4x}$

Integrate to find y: $y = \int 5e^{4x} dx = \frac{5}{4}e^{4x} + c$

To find c, use (2, 6e⁸): $6e^8 = \frac{5}{4}e^8 + c$

$c = \frac{19}{4}e^8$

So the curve is $y = \frac{5}{4}e^{4x} + \frac{19}{4}e^8$

E.g. 3 Find the value of $\int_2^3 e^{2x}\,dx$.

Working:

$$\begin{aligned}\int_2^3 e^{2x}\,dx &= \left[\frac{1}{2}e^{2x}\right]_2^3 \\ &= \frac{1}{2}\left(e^6 - e^4\right) \\ &= \frac{1}{2}e^4\left(e^2 - 1\right)\end{aligned}$$

E.g. 4 Express $\int_3^{15} \frac{2}{x}\,dx$ in the form $\ln A$ where A is to be found.

Working:

$$\begin{aligned}\int_3^{15} \frac{2}{x}\,dx &= \left[2\ln x\right]_3^{15} \\ &= 2\ln 15 - 2\ln 3 \\ &= 2(\ln 15 - \ln 3) \\ &= 2\ln 5 && \textit{by 2nd law of logs} \\ &= \ln 25 && \textit{by 3rd law of logs}\end{aligned}$$

So $A = 25$

Video: Integrating e^x

Video: Integrating reciprocal functions

Solutions to Starter and E.g.s

Exercise
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