Integration of e^x and $\dfrac{1}{x}$

Starter

1.   Given that $\dfrac{d(e^x)}{dx} = e^x$ and $\dfrac{d(\ln x)}{dx} = \dfrac{1}{x}$, find:

   (a) $\displaystyle\int e^x dx$ (b) $\displaystyle\int \dfrac{1}{x}dx$ (c) $\displaystyle\int A e^{kx} dx$ (d) $\displaystyle\int \dfrac{P}{Qx}dx$

   Working: (a) $e^x + c$ (b) $\ln|x| + c$

   (c) $\dfrac{A}{k}e^{kx} + c$ (d) $\dfrac{P}{Q}\ln|x| + c$

E.g. 1 Find: (a) $\displaystyle\int 8e^{3x} dx$ (b) $\displaystyle\int \dfrac{1}{9}e^{4x} dx$ (c) $\displaystyle\int \dfrac{1}{8x}dx$ (d) $\displaystyle\int \dfrac{3}{7x}dx$

   Working: (a) $\dfrac{8}{3}e^{3x} + c$

   (b) $\dfrac{1}{36}e^{4x} + c$

   (c) $\displaystyle\int \dfrac{1}{8x}dx = \dfrac{1}{8}\int \dfrac{1}{x}dx = \dfrac{1}{8}\ln|x| + c$

   (d) $\displaystyle\int \dfrac{3}{7x}dx = \dfrac{3}{7}\int \dfrac{1}{x}dx = \dfrac{3}{7}\ln|x| + c$

E.g. 2 Find the equation of the curve that has derivative $5e^{4x}$ given that the curve passes through the point (2, 6e⁸). Give your answer exactly.

   Working: The derivative is derivative $5e^{4x}$ so $\dfrac{dy}{dx} = 5e^{4x}$

   Integrate to find y: $y = \displaystyle\int 5e^{4x} dx = \dfrac{5}{4}e^{4x} + c$

   To find c, use (2, 6e⁸): $6e^8 = \dfrac{5}{4}e^8 + c$

   $c = \dfrac{19}{4}e^8$

   So the curve is $y = \dfrac{5}{4}e^{4x} + \dfrac{19}{4}e^8$

E.g. 3 Find the value of $\displaystyle\int_2^3 e^{2x}dx$.

Working:
$$\int_2^3 e^{2x}dx = \left[\frac{1}{2}e^{2x}\right]_2^3$$
$$= \frac{1}{2}\left(e^6 - e^4\right)$$
$$= \frac{1}{2}e^4\left(e^2 - 1\right)$$

E.g. 4 Express $\displaystyle\int_3^{15} \frac{2}{x}dx$ in the form $\ln A$ where A is to be found.

Working:
$$\int_3^{15} \frac{2}{x}dx = \left[2\ln x\right]_3^{15}$$
$$= 2\ln 15 - 2\ln 3$$
$$= 2(\ln 15 - \ln 3)$$
$$= 2\ln 5 \qquad \textit{by 2nd law of logs}$$
$$= \ln 25 \qquad \textit{by 3rd law of logs}$$
$$\text{So } A = 25$$

Video: Integrating e^x

Video: Integrating reciprocal functions

Solutions to Starter and E.g.s

Exercise
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