Introduction to Differential Equations

Starter

1. (Review of AS material)

A curve passes through the point $(2, -5)$ and satisfies $\frac{dy}{dx} = 6x^2 - 1$. Find $f(x)$.

Working: $y = \int (6x^2 - 1)dx = 2x^3 - x + c$

Substitute $(2, -5)$: $-5 = 2 \times 2^3 - 2 + c \quad \Rightarrow \quad c = -19$

$y = 2x^3 - x - 19$

2. (Review of AS material)

Given that the gradient function of $f(x)$ is $15x^2 - 6x + 4$ and $f(1) = 0$, find $f(x)$.

Working: $f(x) = \int (15x^2 - 6x + 4)dx = 5x^3 - 3x^2 + 4x + c$

$f(1) = 0$: $\quad 0 = 5 - 3 + 4 + c \quad \Rightarrow \quad c = -6$

$f(x) = 5x^3 - 3x^2 + 4x - 6$

E.g. 1 Find the general solution of the differential equation $\frac{dy}{dx} = \sin 3x - e^{6x} + \sec^2 x$.

Working: $y = \frac{1}{3}\cos 3x - \frac{1}{6}e^{6x} + \tan x + c$

E.g. 2 Find the particular solution of the differential equation $\frac{dy}{dx} = \sqrt{5x + 1}$ given that when $x = 7, y = 8$.

Working: $y = \int \sqrt{5x + 1}\,dx = \int (5x + 1)^{\frac{1}{2}}dx = \frac{2}{15}(5x + 1)^{\frac{3}{2}} + c$

When $x = 7, y = 8$: $\quad 8 = \frac{2}{15}(5 \times 7 + 1)^{\frac{3}{2}} + c \quad \Rightarrow \quad c = -\frac{104}{5}$

$\therefore y = \frac{2}{15}(5x + 1)^{\frac{3}{2}} - \frac{104}{5} = \frac{2}{15}(5x + 1)^{\frac{3}{2}} - 20.8$

Video: Finding the constant term

Solutions to Starter and E.g.s

Exercise

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