

Inverse Normal distribution

Starter

1. The heights of a group of people are normally distributed with a mean of 174 cm and a standard deviation of 6 cm. Find the probability that a person selected at random:
- is at least 170 cm tall
 - is no taller than 180 cm
 - is at least 178 cm given that they are between 172 and 182 cm.

Working:

(a) $X \sim N(174, 6^2)$
 $P(X \geq 170) = 0.7475$
 The probability the person is at least 170 cm tall is 0.7475 (4 s.f.)

(b) $P(X \leq 180) = 0.8413$
 The probability the person is no taller than 180 cm is 0.8413 (4 s.f.)

(c)
$$P(X \geq 178 | 172 < X < 182) = \frac{P(X \geq 178 \cap 172 < X < 182)}{P(172 < X < 182)}$$

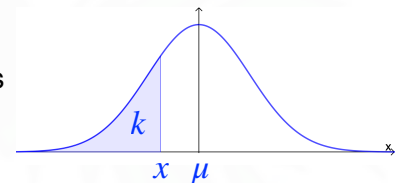
$$= \frac{P(178 \leq X < 182)}{P(172 < X < 182)}$$

$$= \frac{0.16128}{0.53935}$$

$$\approx 0.2990$$
 The probability is at least 178 cm given that they are between 172 and 182 cm is 0.2990 (4 s.f.)

Considering areas diagrammatically

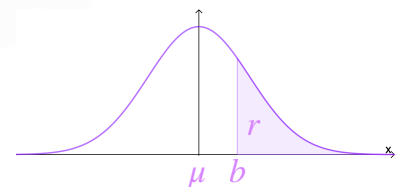
E.g. 1 The inverse normal function of a modern calculator calculates the x -value when the area is similar to the one in the diagram $P(X < x) = k$.



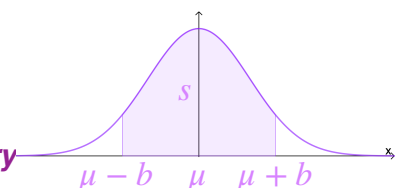
- Sketch a diagram and shade the area represented by r for $P(X > b) = r$. Write down a formula for $P(X < b)$ involving r .
- Sketch a diagram and shade the area represented by s for $P(|X - \mu| < b) = s$. Write down a formula for $P(X < \mu + b)$ involving s , but not $P(X < \mu - b)$.
- Sketch a diagram and shade the area represented by t for $P(a < X < b) = t$, where $a < \mu < b$. If $P(X < a)$ is given, find a formula for $P(X < b)$ in terms of t and $P(X < a)$.

Working:

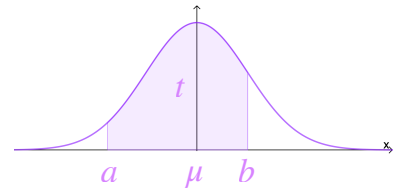
(a) $P(X > b) = r$
 $P(X < b) = 1 - r$
since area under normal curve is 1



(b) $P(|X - \mu| < b) = s$
 $P(\mu - b < X < \mu + b) = s$
 $P(\mu < X < \mu + b) = \frac{s}{2}$ *symmetry*
 $P(X < \mu + b) = \frac{s}{2} + 0.5$ *since $P(X < \mu) = 0.5$*



(c) $P(a < X < b) = t$
 $P(X < b) = t + P(X < a)$



E.g. 2 Given that $X \sim N(52, 40)$, find p, q, r and s correct to 3 s.f. such that:

- (a) $P(X < p) = 0.647$ (b) $P(X > q) = 0.581$
 (c) $P(52 < X < r) = 0.3$ (d) $P(|X - 52| < s) = 0.36$

- Working:**
- (a) $P(X < p) = 0.647 \Rightarrow p = 53.4$ (3 s.f.)
- (b) $P(X > q) = 0.581 \Rightarrow P(X < q) = 1 - 0.581 = 0.419$
 $\therefore q = 50.7$ (3 s.f.)
- (c) $P(52 < X < r) = 0.3$
 Since $\mu = 52, P(X < 52) = 0.5 \Rightarrow P(X < r) = 0.8$
 $\therefore r = 57.3$ (3 s.f.)
- (d) $P(|X - 52| < s) = 0.36$ is the same as
 $P(52 - s < X < 52 + s) = 0.36$ *symmetry about the mean*
 $P(52 < X < 52 + s) = 0.18$
 $\Rightarrow P(X < 52 + s) = 0.68$ *since $P(X < 52) = 0.5$*
 $52 + s = 54.958 \Rightarrow s = 2.96$ (3 s.f.)

E.g. 3 Given that $y \sim N(20, 2^2)$, find a, b, c and d correct to 3 s.f. such that:

- (a) $P(X > a) = 0.28$ (b) $P(X < b) = 0.183$
 (c) $P(|X - 20| < c) = 0.84$ (d) $P(18 < X < d) = 0.45$

- Working:**
- (a) $P(X > a) = 0.28 \Rightarrow P(X < a) = 1 - 0.28 = 0.72$
 $\therefore a = 21.2$ (3 s.f.)
- (b) $P(X < b) = 0.183 \Rightarrow b = 18.2$ (3 s.f.)
- (c) $P(|X - 20| < c) = 0.84$ is the same as
 $P(20 - c < X < 20 + c) = 0.84$ *symmetry about the mean*
 $P(20 < X < 20 + c) = 0.42$
 $\Rightarrow P(X < 20 + c) = 0.92$ *since $P(X < 20) = 0.5$*
 $20 + c = 22.81 \Rightarrow c = 2.81$ (3 s.f.)
- (d) $P(18 < X < d) = 0.45$
 $P(X < 18) = 0.158655 \Rightarrow P(X < d) = 0.608655$
 $\therefore d = 20.6$ (3 s.f.)

E.g. 4 The board of examiners have decided that 80 % of all candidates sitting A level maths will obtain a pass grade. The actual exam marks are found to be normally distributed with a mean of 45 and a standard deviation of 7.

- (a) What is the lowest score a student can get on the exam to be awarded a pass grade?
(b) Given that 10 % of students will achieve an A*, calculate the lowest mark required to get the highest grade.

Give your answers to the nearest mark.

Working: (a) $X \sim N(45, 7^2)$
Let p be the pass mark: $P(X > p) = 0.8$
 $P(X < p) = 1 - 0.8 = 0.2 \Rightarrow p = 39.1$
The pass mark is 39.

(b) Let a be the mark required to get an A*: $P(X > a) = 0.1$
 $P(X < a) = 1 - 0.1 = 0.9 \Rightarrow a = 54.0$
The mark required to achieve an A* is 54.

E.g. 5 Batteries for a radio have a mean life of 160 hours and a standard deviation of 30 hours. Assuming the battery life follows a normal distribution, calculate:

- (a) the proportion of batteries which have a life between 150 and 180 hours
(b) the range, symmetrical about the mean, within which 75 % of batteries lie.

Working: (a) $X \sim N(160, 30^2)$
 $P(150 < X < 180) = 0.378$
The proportion of batteries which have a life between 150 and 180 hours is 0.378.

(b) $P(160 - x < X < 160 + x) = 0.75$ *symmetry about the mean*
 $P(160 < X < 160 + x) = 0.375$
 $\Rightarrow P(X < 160 + x) = 0.875$ *since $P(X < 160) = 0.5$*
 $160 + x = 194.5 \Rightarrow x = 34.5$
The range is 125.5 to 194.5 hours.

Video: [Inverse normal function](#)

[Normal distribution EQ](#)

[Solutions to Starter and E.g.s](#)

Exercise

p386 17C Qu 1i, 2-7, (8-11 red)