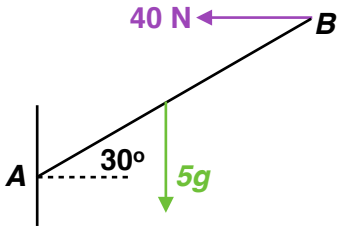
Ladder Problems

Starter

1. **(Review of last lesson)**

A non-uniform beam, AB, of mass 5 kg and length b m, is freely hinged to a vertical wall at A. The beam is held in equilibrium at an angle of 30^o to the horizontal by a force of magnitude 40 N, applied horizontally at B, as shown. Find the distance of the centre of mass of the beam from A. Give your answer in terms of b.



Working: Let d be the distance of the centre of mass of the beam from A

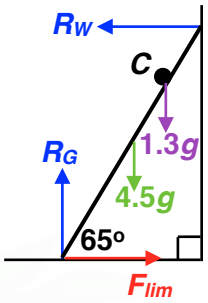
↻ about A: $5g\cos 30 \times d = 40\sin 30 \times b$

$$d = \frac{40\sin 30 \times b}{5g\cos 30}$$

$$d \approx 0.471b \text{ m}$$

The distance of the centre of mass of the beam from A is $0.471b$ (3 s.f.).

E.g. 1 A ladder rests against a smooth vertical wall at an angle of 65^o to rough horizontal ground. The ladder has mass 4.5 kg and length $5x$ m. A cat of mass 1.3 kg sits on the ladder at C, $4x$ m from the base. The ladder is in limiting equilibrium. Modelling the ladder as a uniform rod and the cat as a particle, find the coefficient of friction between the ground and the ladder.



Working: $F_{limG} = \mu R_G$ so we need to find F_{limG} and R_G.

↻ about R_G: $4.5g\cos 65 \times 2.5x + 1.3g\cos 65 \times 4x = R_W \cos 25 \times 5x$

Dividing by x: $11.25g\cos 65 + 5.2g\cos 65 = 5\cos 25 \times R_W$

$$R_W = \frac{11.25g\cos 65 + 5.2g\cos 65}{5\cos 25}$$

$$R_W = 15.034$$

$R(\rightarrow)$: $F_{limG} = R_W = 15.034\ldots$ N

$R(\uparrow)$: $R_G = 4.5g + 1.3g = 56.84$ N

$F_{lim} = \mu R$: $\mu = \dfrac{F_{limG}}{R_G} = \dfrac{15.034}{56.84}$

$$\mu = 0.26$$

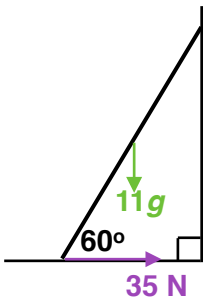
E.g. 2 A uniform ladder of mass 11 kg and length 7 m rests against a rough vertical wall, at an angle of 60^o to smooth, horizontal ground, as shown. A horizontal force of magnitude 35 N is applied to the base of the ladder, keeping it in limiting equilibrium, with the ladder on the point of sliding up the wall. Find:

(a) the magnitude of the normal reaction of the wall on the ladder,

(b) the frictional force between the wall and the ladder,

(c) the coefficient of friction between the wall and the ladder.

Hint: draw a diagram with all the forces included.

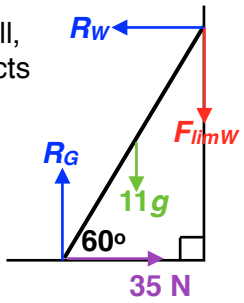


Working:

(a) Since the ladder is on the point of sliding up the wall, the friction force between the ladder and the wall acts downwards.

$R(\rightarrow)$: $\quad R_W = 35$

The magnitude of the normal reaction of the wall on the ladder is 35 N



(b) $\circlearrowleft$ about foot of ladder:

$$F_{limW} \cos 60 \times 7 + 11g \cos 60 \times 3.5 = R_W \sin 60 \times 7$$

Since $R_W = 35$:

$$F_{limW} \cos 60 \times 7 = 35 \sin 60 \times 7 - 11g \cos 60 \times 3.5$$

$$F_{limW} = \frac{35 \sin 60 \times 7 - 11g \cos 60 \times 3.5}{\cos 60 \times 7}$$

$$F_{limW} \approx 6.72 \text{ N}$$

The frictional force between the wall and the ladder is 6.72 N (3 s.f.)

(c) $F_{limW} = \mu R_W$: $\quad \mu = \dfrac{6.72...}{35}$ $\qquad$ $6.72...$ ***is value from calc***

$$\mu \approx 0.192$$

The coefficient of friction between the wall and the ladder is 0.192 (3 s.f.).

Video: Ladder problems

Ladder problems EQ

Solutions to Starter and E.g.s

Exercise

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Additional questions

1. A uniform ladder of mass 10 kg and length 6 m rests with one end on rough, horizontal ground and the other end against a smooth, vertical wall. The coefficient of friction between the ground and the ladder is 0.3, and the ladder makes an angle of 65^o with the ground. A girl of mass 50 kg begins to climb the ladder. How far up the ladder can she climb before the ladder slips?

2. A uniform ladder of mass 9 kg and length 4.8 m rests in limiting equilibrium with one end on rough horizontal ground and the other end against a rough, vertical wall. The normal reactions at the wall and the ground are 22 N and 75 N respectively. Find:
   (a) the angle that the ladder makes with the ground,
   (b) the coefficient of friction between the wall and the ladder,
   (c) the coefficient of friction between the ground and the ladder.

3*. Robert holds an 8 m uniform ladder in place against a smooth vertical wall by applying a horizontal force of K N to it, 1 m from the base of the ladder. The ladder weighs 100 N and makes an angle of 75° with the rough horizontal floor, where $\mu = 0.1$. Given that the ladder remains at rest, find the range of possible values for the magnitude of the force K.

Answers to additional questions

1. 4.03 m

2. (a) 54.6°
   (b) 0.6
   (c) 0.29

3. $3.882\ldots \leq K \leq 26.73\ldots$