# Locating Roots of Functions

**Starter**

1. **(Review of last lesson)** Express $72^o$ as an angle in radians in terms of $\pi$.

***Working:*** $72^o \equiv 72 \times \frac{\pi}{180} = \frac{2\pi}{5}$

2. **(Review of last lesson)** Convert $\frac{5\pi}{8}$ to an angle in radians.

***Working:*** $\frac{5\pi}{8} \equiv \frac{5\pi}{8} \times \frac{180}{\pi} = 112.5^o$

***E.g. 1*** Find the two consecutive integers between which the root of the equation $x^3 + x = 20$ lies.

***Working:***

***Rearrange to make it equal*** $0$***:*** $x^3 + x - 20 = 0$

***Replace*** $0$ ***by*** $f(x)$***:*** $f(x) = x^3 + x - 20$

***Now use trial and improvement:***

$f(1) = 1^3 + 1 - 20 < 0$

$f(2) = 2^3 + 2 - 20 < 0$

$f(3) = 3^3 + 3 - 20 > 0$

Since there is a sign change, there is a root between $2$ and $3$.

***E.g. 2*** Show that one root of the equation $x^3 - x^2 - 9 = 0$ is $2.472$, correct to 3 d.p.

***Working:***

Let $f(x) = x^3 - x^2 - 9$

Lower bound of $2.472$ is $2.4715$: $f(2.4715) < 0$

Upper bound of $2.472$ is $2.4725$: $f(2.4725) > 0$

Since there is a sign change between the upper and lower bounds of $2.472$, $x = 2.472$ is solution to 3 d.p. of the equation $x^3 - x^2 - 9 = 0$

***N.B.*** In questions with trigonometry, use radians.

***E.g. 3*** Evaluate $f(x) = x^4 - 7x^2 + 3x + 4$ for integer values of $x$ from $-3$ to $3$. Use your table to give integer bounds for the roots of the function.

**Working:**

| **x** | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| **y** | $13$ | $-14$ | $-5$ | $4$ | $1$ | $-2$ | $31$ |

Given that the curve is continuous there are roots:

between $-3$ and $-2$

between $-1$ and $0$

between $1$ and $2$

between $2$ and $3$

***E.g. 4*** Show that there is a solution to $e^{3x} \sin x = 5$ in the interval $(0, 1)$.

***Working:*** Let $f(x) = e^{3x} \sin x - 5$

***Your calculator must be in radians because we are using trigonometry.***

$f(1) = e^{3\times1} \times \sin 1 - 5 \approx 11.9 > 0$

$f(2) = e^{3\times2} \times \sin 2 - 5 \approx 362 > 0$

It looks like I am going in the wrong direction…

$f(0) = e^{0} \times \sin 0 - 5 = -5 < 0$

Since the curve is continuous and there is a sign change between $0$ and $1$, there is a solution to $e^{3x} \sin x = 5$ between $x = 0$ and $x = 1$

***E.g. 5*** Let $f(x) = e^x - x^3 - 5x$. Verify that a solution of the equation $f(x) = 0$ is $x = 0.25$ correct to 2 decimal places.

***Working:*** Lower bound of $0.25$ is $0.245$: $f(0.245) > 0$

Upper bound of $0.25$ is $0.255$: $f(0.255) < 0$

Since there is a sign change between the lower and upper bounds of $0.25$, the root of $0.25$ is correct to 2 decimal places.

Video: Change of sign method

Solutions to Starter and E.g.s

**Exercise**

*N.B. If there is a trigonometric ratio, calculators need to be in radians.*

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