Modelling Projectile Motion

Starter

1. **(Review of AS material)** A particle is projected vertically upward from ground level with a speed of 24.5 m/s. Find:
   (a) the greatest height reached
   (b) the time that elapses before the particle returns to the ground.

Working:

(a) At the greatest height $v = 0$
So $u = 24.5, v = 0, a = -9.8, s = ?$
No $t \quad \Rightarrow \quad v^2 = u^2 + 2as$: $\quad 0 = 24.5^2 + 2 \times -9.8 \times s$
$s = 30.6$ so the greatest height is 30.6 m

(b) When the particle returns to ground $s = 0$
So $u = 24.5, a = -9.8, s = 0, t = ?$
No $v \quad \Rightarrow \quad s = ut + \frac{1}{2}at^2$: $\quad 0 = 24.5t + \frac{1}{2} \times (-9.8) \times t^2$

$t(24.5 - 4.9t) = 0 \quad \Rightarrow \quad t = 0$ or $t = \frac{24.5}{4.9} = 5$

$\therefore$ the time to return to ground is 5 seconds

2. **(Review of AS material)** A particle starts from the origin and moves under acceleration $\mathbf{a} = (2\mathbf{i} - 7\mathbf{j})$ m/s². Given that the initial velocity is $\mathbf{u} = (-\mathbf{i} + 6\mathbf{j})$. Find:
   (a) the position and velocity of the particle after 4 s and
   (b) the time when the particle is at the point $(\lambda\mathbf{i} - 12\mathbf{j})$, where λ is a value to be found.

Working:

(a) Position: $\quad \mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = (-\mathbf{i} + 6\mathbf{j}) \times 4 + 0.5 \times (2\mathbf{i} - 7\mathbf{j}) \times 4^2$
$\mathbf{s} = (12\mathbf{i} - 32\mathbf{j})$ m

Velocity: $\quad \mathbf{v} = \mathbf{u} + \mathbf{a}t$
$\mathbf{v} = (-\mathbf{i} + 6\mathbf{j}) + (2\mathbf{i} - 7\mathbf{j}) \times 4$
$\mathbf{v} = (7\mathbf{i} - 22\mathbf{j})$ m/s

(b) $s = ut + \frac{1}{2}at^2$

$\mathbf{j}$: $\quad -12 = 6t + 0.5 \times (-7) \times t^2$
$t = 2.90$ or $t = -1.18$

$\mathbf{i}$: $\quad$ Since $t > 0$, when $t = 2.90$,
$\lambda = -1 \times 2.90 + 0.5 \times 2 \times 2.90^2 = 5.51$

E.g. 1 Find the horizontal and vertical components of the velocity when a particle is moving with speed 8 m/s at an angle of 35° above the horizontal.

Working:

Horizontal $= 8\cos 35 = 6.55$ m/s; $\quad$ ***turn through 35° so cos***
Vertical $= 8\sin 35 = 4.59$ m/s $\quad$ ***not turn through 35° so sin***

E.g. 2 A ball is thrown with velocity v, with horizontal component 6 m/s and vertical component 8 m/s. Find the speed and direction of projection of the ball.

Working: Speed $= \sqrt{6^2 + 8^2} = 10$ m/s;

Direction $= \tan^{-1}\frac{8}{6} = 53.1^{\circ}$ above the horizontal

E.g. 3 A particle is projected with initial speed 18 m/s at an angle of 40° above the horizontal. Find the distance from its point of projection 1.8 s after it is projected.

Working: ***The aim is to find the distance travelled horizontally and vertically and then use Pythagoras.***

$u_x = 18\cos 40,\ a_x = 0,\ t = 1.8,\ s_x = ?$ (horizontally)

No v_x $\Rightarrow$ $s = ut + \frac{1}{2}at^2$

$s_x = 18 \times 1.8 \times \cos 40 + 0 = 24.8\ldots$

$u_y = 18\sin 40,\ a_y = -9.8,\ t = 1.8,\ s_y = ?$ (vertically)

No v_y $\Rightarrow$ $s = ut + \frac{1}{2}at^2$

$s_y = 18 \times 1.8 \times \sin 40 - 0.5 \times 9.8 \times 1.8^2 = 13.87\ldots$

Distance $= \sqrt{24.8^2 + 13.87^2} = 28.4$ m (3 s.f.)

E.g. 4 A particle is projected from the point $5\mathbf{j}$. The particle's initial velocity is $(7\mathbf{i} + 3\mathbf{j})$ m/s and it moves freely under gravity.

(a) Find its position vector 0.8 s after it is projected.

(b) Find the velocity at this time.

Working: (a) $\mathbf{u} = 7\mathbf{i} + 3\mathbf{j},\ t = 0.8,\ \mathbf{a} = -9.8\mathbf{j},\ \mathbf{s} = ?$

No $\mathbf{v}$ $\Rightarrow$ $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

The starting position is $5\mathbf{j}$ so:

$\mathbf{s} = 5\mathbf{j} + (7\mathbf{i} + 3\mathbf{j}) \times 0.8 + \frac{1}{2} \times -9.8\mathbf{j} \times 0.8^2$

$= (5.6\mathbf{i} - 4.264\mathbf{j})$

The position vector after 0.8 s is $(5.6\mathbf{i} + 4.264\mathbf{j})\mathbf{m}$

(b) $\mathbf{u} = 7\mathbf{i} + 3\mathbf{j},\ t = 0.8,\ \mathbf{a} = -9.8\mathbf{j},\ \mathbf{v} = ?$

No $\mathbf{s}$ $\Rightarrow$ $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{v} = (7\mathbf{i} + 3\mathbf{j}) + (0\mathbf{j} - 9.8\mathbf{j}) \times 0.8 = (7\mathbf{i} - 4.84\mathbf{j})$ m/s

Video: Projectiles

Video: Projectiles (upward angle/above ground)

Video: Projectiles (downward angle/above ground)

Video: Projected horizontally

Solutions to Starter and E.g.s

Exercise

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Answers to additional questions

1. Horizontal $= 6.11\cos\alpha$ m/s; vertical $= 6.11\sin\alpha$ m/s

2. (a) $v_x = 10, v_y = -4.9$ so speed $= 11.1$ m/s (3 s.f.)
   and direction = 26.1° (3 s.f.) below horizontal
   (b) $s_y = -2, t = 0.6388\ldots$ so $s_x = 6.39$

3. (a) $(12\mathbf{i} - 3.6\mathbf{j})$ m/s
   (b) $12\mathbf{i}$ m/s
   (c) $(12\mathbf{i} - 16\mathbf{j})$ m/s

4. (a) 10.1 m
   (b) 22.1 m/s
   (c) 39.6° below the horizontal

5. (a) 0.719 s
   (b) 7.51 m

6. (a) $a = 40, b = 16.5$
   (b) $\mathbf{v} = (40\mathbf{i} - 32.5\mathbf{j})$ m/s