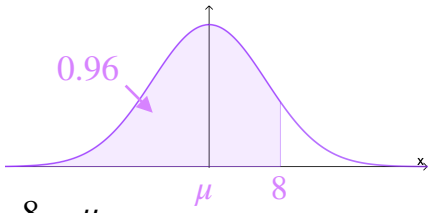
Modelling with the Normal distribution

Starter

1. The thickness of some sheets of wood follows a normal distribution with mean μ and standard deviation σ. $96\,\%$ of the sheets will go through an 8 mm gauge while only $1.7\,\%$ will go through a 7 mm gauge. Find μ and σ.

Working: $96\,\%$ of sheets will go through an 8 mm gauge



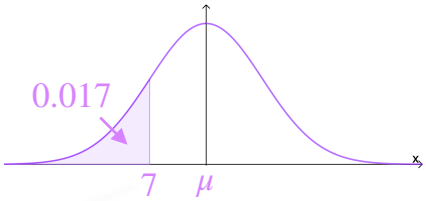
Let z_1 be the $z-$value corresponding to $x = 8$

$P(Z < z_1) = 0.96 \quad \Rightarrow \quad z_1 = 1.75$

Substitute into $z = \dfrac{x-\mu}{\sigma}$: $\quad 1.75 = \dfrac{8-\mu}{\sigma}$

$\mu + 1.75\sigma = 8$

$1.7\,\%$ will go through a 7 mm gauge



Let z_2 be the $z-$value corresponding to $x = 7$.

$P(Z < z_1) = 0.017 \quad \Rightarrow \quad z_2 = -2.12$

Substitute into $z = \dfrac{x-\mu}{\sigma}$: $\quad -2.12 = \dfrac{7-\mu}{\sigma}$

$\mu - 2.12\sigma = 7$

Solving simultaneously: $\quad \mu \approx 7.55, \sigma \approx 0.258$ (3 s.f.)

E.g. 1 A fair coin is tossed 12 times and the number of heads is counted. Decide whether this can be approximated by the normal distribution.

Working: $n = 12, p = 0.5$

$np = 12 \times 0.5 = 6 > 5$

$n(1-p) = 12 \times 0.5 = 6 > 5$

Since $np > 5$ and $n(1-p) > 5$, it can be accurately approximated by the normal distribution.

E.g. 2 Using a normal approximation to the binomial distribution, find the probability of obtaining more than 110 ones in 400 tosses of an unbiased tetrahedral dice with faces marked 1, 2, 3 and 4.

Working: $n = 400, p = 0.25 \quad \Rightarrow \quad X \sim \text{B}\,(400, 0.25)$

$np = 100 \qquad np(1-p) = 75$

$X \sim \text{B}\,(400, 0.25) \quad \approx \quad X \sim \text{N}\,(100, 75)$

$P(X > 110) = 0.1241$

E.g. 3 It is known that in a sack of mixed grass seeds $35\ \%$ are ryegrass. Let X be the distribution of the number of ryegrass in a sack.

(a) Find the smallest number of seeds that can be in the sack for the distribution to be approximated by the normal distribution.

Given that there are 300 seeds in the sack, and using a normal approximation, find the probability that there are:

(b) less than 120 ryegrass seeds,

(c) between 120 and 150 ryegrass seeds (inclusive)

Working:

(a) Let n be the smallest number of seeds.

$np > 5$: $\quad 0.35n > 5 \quad \Rightarrow \quad n > 14.3$

$n(1-p) > 5$: $\quad 0.65n > 5 \quad \Rightarrow \quad n > 7.7$

The smallest number of seeds is 15.

(b) $n = 300, p = 0.35 \quad \Rightarrow \quad X \sim \text{B}(300, 0.35)$

$np = 105 \qquad np(1-p) = 68.25$

$X \sim \text{B}(300, 0.35) \quad \approx \quad X \sim \text{N}(105, 68.25)$

$P(X < 120) = 0.9653$

(c) $P(120 \leq X \leq 150) = 0.03471$

Video: Modelling using the Normal distribution

Video: Normal approximation to binomial distribution

Solutions to Starter and E.g.s

Exercise

p393 17E Qu 1-5, (6-8 red)