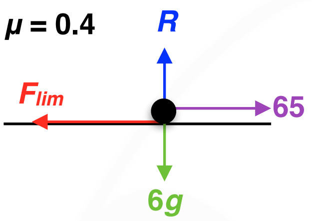
Motion on a Slope

Starter

1. **Horizontal plane**

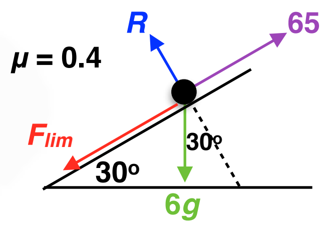
(Review of last lesson)

A mass of 6 kg is pulled along a horizontal plane by a horizontal force of 65 N. The coefficient of friction between the particle and surface is 0.4. Find the acceleration of the particle.



2. **Inclined plane**

What will the acceleration be if the plane is inclined at an angle of 30^{o} and the force of 65 N acts parallel and up the slope?



Working: 1.

$\mu = 0.4$

$R(\uparrow)$: $\quad R = 6g$

$F_{lim} = \mu R$: $\quad F_{lim} = 0.4 \times 6g = 2.4g$

$F = ma(\rightarrow)$: $\quad 65 - F_{lim} = 6a$

$65 - 2.4g = 6a$

$a = 6.91$ m/s² (3 s.f.)

2.

$R(\perp)$: $\quad R = 6g\cos 30 = 3\sqrt{3}g$

$F_{lim} = \mu R$: $\quad F_{lim} = 0.4 \times 3\sqrt{3}g = 1.2\sqrt{3}g$

$F = ma(//)$: $\quad 65 - F_{lim} - 6g\sin 30 = 6a$

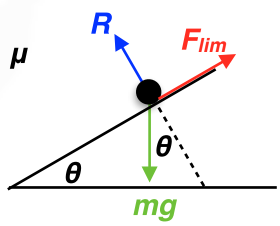
N.B. The $-6g\sin 30$ is from the ***weight*** acting down the incline.

$65 - 1.2\sqrt{3}g - 6g\sin 30 = 6a$

$a = 2.54$ m/s² (3 s.f.)

E.g. 1 Accelerating down the plane under its own weight

Mass = m kg, plane inclined at θ, acceleration = a



Working: Friction is resisting motion down the slope.

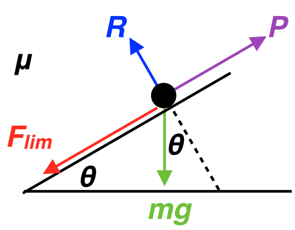
$R(\perp)$: $\quad R = mg\cos\theta$

$F_{lim} = \mu R$: $\quad F_{lim} = \mu mg\cos\theta$

$F = ma(//)$: $\quad mg\sin\theta - F_{lim} = ma$

E.g. 2 Accelerating up the plane under the action of force P acting parallel to the plane

Mass = m kg, plane inclined at θ, acceleration = a



Working: Friction is resisting motion up the slope.

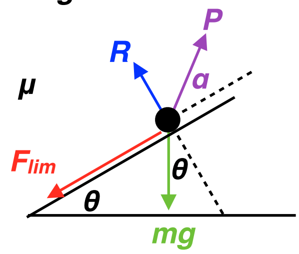
$R(\perp)$: $\quad R = mg\cos\theta$

$F_{lim} = \mu R$: $\quad F_{lim} = \mu mg\cos\theta$

$F = ma(//)$: $\quad P - mg\sin\theta - F_{lim} = ma$

E.g. 3 ***Accelerating up the plane under the action of force P acting at an angle of α to the plane***

Mass = m kg, plane inclined at θ, acceleration = a



Working: Friction is resisting motion up the slope.

$R(\perp)$: $R + P\sin\alpha = mg\cos\theta$

$R = mg\cos\theta - P\sin\alpha$

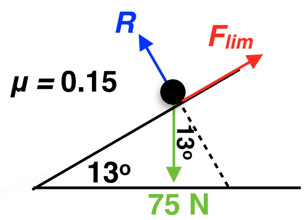
$F_{lim} = \mu R$: $F_{lim} = \mu(mg\cos\theta - P\sin\alpha)$

$F = ma(//)$: $P - mg\sin\theta - F_{lim} = ma$

E.g. 4 A snow-covered hill is at an angle of 13^{o} to the horizontal. A toboggan of weight 75 N is placed on the hill. Given that the coefficient of friction between the toboggan and the hill is 0.15, find whether the toboggan will slide down the hill by itself. If so, calculate the acceleration.

Working: $R(\perp)$: $R = 75\cos 13$

$F_{lim} = \mu R$: $F_{lim} = 0.15 \times 75\cos 13$
$= 11.25\cos 13$



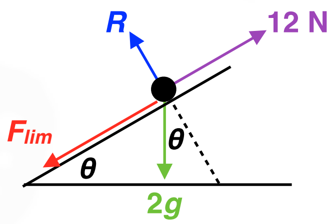
$F = ma(//)$: $75\sin 13 - F_{lim} = \dfrac{75}{g}a$

$$75\sin 13 - 11.25\cos 13 = \frac{75}{g}a$$

Since $75\sin 13 > 11.25\cos 13$, the toboggan will slide down the hill.
The acceleration is 0.772 m/s² (3 s.f.)

E.g. 5 A box of mass 2 kg is at rest on a rough plane inclined at an angle of θ to the horizontal. A force of 12 N acts up the plane on the box, which is on the point of moving up the slope. Given that $\cos\theta = \dfrac{4}{5}$, find the coefficient of friction between the box and the plane to 3 s.f.

Working: If $\cos\theta = \dfrac{4}{5}$, then $\sin\theta = \dfrac{3}{5}$ and $\tan\theta = \dfrac{3}{4}$



$R(\perp)$: $R = 2g\cos\theta = \dfrac{8}{5}g$

$F_{lim} = \mu R$: $F_{lim} = \dfrac{8}{5}\mu g$

There is no acceleration so we could resolve up the plane.

Either $R(//)$: $12 = 2g\sin\theta + F_{lim}$

$$F_{lim} = 12 - \frac{6}{5}g$$

or $F = ma(//)$: $12 - 2g\sin\theta - F_{lim} = 0$

$$F_{lim} = 12 - \frac{6}{5}g$$

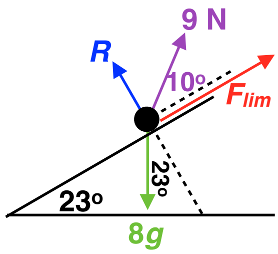
Substitute into $F_{lim} = \dfrac{8}{5}\mu g$: $12 - \dfrac{6}{5}g = \dfrac{8}{5}\mu g$

$$\mu \approx 0.0153$$

The coefficient of friction between the box and the plane 0.0153 (3 s.f.).

E.g. 6 A stone of mass 8 kg is at rest on a rough slope inclined at 23° to the horizontal. A force of magnitude 9 N acts on the box at an angle of 10° to the slope. Calculate the coefficient of friction given that the 9 N force is just enough to stop the stone sliding down the slope.

Working: The force is just enough to stop the stone ***sliding down the slope*** so ***friction acts up the slope***.



$R(\perp)$: $R + 9\sin 10 = 8g\cos 23$

$R = 8g\cos 23 - 9\sin 10$

$F_{lim} = \mu R$: $F_{lim} = \mu(8g\cos 23 - 9\sin 10)$

Either $R(//)$: $9\cos 10 + F_{lim} = 8g\sin 23$

$F_{lim} = 8g\sin 23 - 9\cos 10$

or $F = ma(//)$: $9\cos 10 + F_{lim} - 8g\sin 23 = 0$

$F_{lim} = 8g\sin 23 - 9\cos 10$

Substitute into $F_{lim} = \mu(8g\cos 23 - 9\sin 10)$:

$8g\sin 23 - 9\cos 10 = \mu(8g\cos 23 - 9\sin 10)$

$$\mu = \frac{8g\sin 23 - 9\cos 10}{8g\cos 23 - 9\sin 10}$$

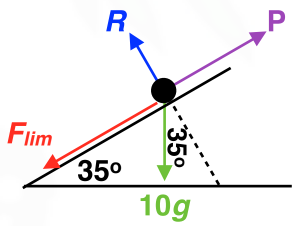
$\mu = 0.308$

The coefficient of friction is 0.308.

E.g. 7* A box of mass 10 kg lies on a rough plane inclined at an angle of 35° to the horizontal. The coefficient of friction is 0.6. A force of P N acts up and parallel to the plane. Given that the box is in limiting equilibrium, calculate the range of values of P.

Working: The box could be just about to move up the slope or down the slope.
If it is ***about to move up*** the slope, ***friction is acting down the slope***.
If it is ***about to move down*** the slope, ***friction is acting up the slope***.

Case 1: about to ***move up*** the slope



$R(\perp)$: $R = 10g\cos 35$

$F_{lim} = \mu R$: $F_{lim} = 0.6 \times 10g\cos 35$
$= 6g\cos 35$

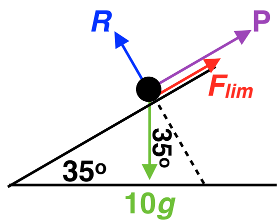
There is no acceleration so we could resolve up the plane.

Either $R(//)$: $P = 10g\sin 35 + F_{lim}$
$= 10g\sin 35 + 6g\cos 35$
$= 104.4$ N

or $F = ma(//)$: $P - F_{lim} - 10g\sin 35 = 0$

$P = 10g\sin 35 + 6g\cos 35$
$= 104.4$ N

Case 2: about to ***move down*** the slope



$R(\perp)$: $R = 10g\cos 35$

$F_{lim} = \mu R$: $F_{lim} = 0.6 \times 10g\cos 35$
$= 6g\cos 35$

There is no acceleration so we could resolve up the plane.

Either $R(//)$: $P + F_{lim} = 10g\sin 35$

$P = 10g\sin 35 - 6g\cos 35$

$= 8.044$ N

or $F = ma(//)$: $P + F_{lim} - 10g\sin 35 = 0$

$P = 10g\sin 35 - 6g\cos 35$

$= 8.044$ N

The range of values of P are $8.044 \leq P \leq 104.4$.

Video: Motion on rough inclined plane

Video: Motion on rough inclined plane example

Motion on rough inclined plane EQ

Solutions to Starter and E.g.s

Exercise

p488 21C Qu 3-8