Partial fractions with distinct factors

Starter

1. **(Review of last lesson)** Without using polynomial division, find the quotient and remainder when $2x^3 - 4x^2 + 3x - 1$ is divided by $x^2 - 1$.

Working: Deg. of quotient = Deg. of dividend — Deg. of divisor $= 3 - 2 = 1$

Quotient is of the form $Ax + B$

Degree of remainder = Degree of divisor $-1 = 2 - 1 = 1$

Remainder is of the form $Cx + D$

$$\frac{2x^3 - 4x^2 + 3x - 1}{x^2 - 1} \equiv Ax + B + \frac{Cx + D}{x^2 - 1}$$

$$2x^3 - 4x^2 + 3x - 1 \equiv (x^2 - 1)(Ax + B) + Cx + D$$

Equating coefficient: x^3: $2 = A$

x^2: $-4 = B$

x: $3 = -A + C$ $\quad \therefore C = 5$

constant: $-1 = -B + D$ $\quad \therefore D = -5$

$$\frac{2x^3 - 4x^2 + 3x - 1}{x^2 - 1} \equiv 2x - 4 + \frac{5x - 5}{x^2 - 1}$$

The quotient is $2x - 4$ and the remainder is $5x - 5$.

2. **(Review of GCSE material)** Simplify: (a) $\frac{x}{4} + \frac{x}{3}$ (b) $\frac{2}{x+1} + \frac{3}{x-4}$

Working: (a) $\frac{x}{4} + \frac{x}{3} = \frac{3x}{12} + \frac{4x}{12} = \frac{7x}{12}$

(b)
$$\frac{2}{x+1} + \frac{3}{x-4} = \frac{2(x-4)}{(x+1)(x-4)} + \frac{3(x+1)}{(x+1)(x-4)}$$
$$= \frac{2x - 8}{(x+1)(x-4)} + \frac{3x + 3}{(x+1)(x-4)}$$
$$= \frac{5x - 5}{(x+1)(x-4)}$$

E.g. 1 Express the following in partial fractions.

(a) $\frac{2x - 1}{(x-1)(x-7)}$ (b) $\frac{3}{x(5x+8)}$

Working: (a) ***Substitution method:***

$$\frac{2x - 1}{(x-1)(x-7)} \equiv \frac{A}{x-1} + \frac{B}{x-7}$$

Multiply by $(x-1)(x-7)$:

$$2x - 1 \equiv A(x-7) + B(x-1)$$

Let $x = 7$: $2 \times 7 - 1 = B(7 - 1)$ $\quad \therefore B = \frac{13}{6}$

Let $x = 1$: $2 \times 1 - 1 = A(1 - 7)$ $\quad \therefore A = -\frac{1}{6}$

So $\frac{2x - 1}{(x-1)(x-7)} \equiv \frac{13}{6(x-7)} - \frac{1}{6(x-1)}$

Equating coefficients method:

$$\frac{2x-1}{(x-1)(x-7)} \equiv \frac{A}{x-1} + \frac{B}{x-7}$$

***Multiply by** $(x-1)(x-7)$:*

$$2x - 1 \equiv A(x-7) + B(x-1)$$

Equating coefficients of x: $2 = A + B$

Equating the constant term: $-1 = -7A - B$

Solving simultaneously gives: $A = -\frac{1}{6}$ $B = \frac{13}{6}$

So $\frac{2x-1}{(x-1)(x-7)} \equiv \frac{13}{6(x-7)} - \frac{1}{6(x-1)}$

(b) $\frac{3}{x(5x+8)} \equiv \frac{A}{x} + \frac{B}{5x+8}$

***Multiply by** $x(5x+8)$:*

$$3 \equiv A(5x+8) + Bx$$

Equating the constant terms: $3 = 8A$ $\therefore A = \frac{3}{8}$

Equating coefficients of x: $0 = 5A + B$ $\therefore B = -\frac{15}{8}$

So $\frac{3}{x(5x+8)} \equiv \frac{3}{8x} - \frac{15}{8(5x+8)}$

E.g. 2 Express the following in partial fractions.

(a) $\frac{7x+4}{5x^2-30x}$ (b) $\frac{x+1}{3x^2-x-2}$

Working: (a) $\frac{7x+4}{5x^2-30x} \equiv \frac{7x+4}{5x(x-6)} \equiv \frac{A}{5x} + \frac{B}{x-6}$

***Multiply by** $5x(x-6)$:*

$$7x + 4 \equiv A(x-6) + 5Bx$$

Let $x = 6$: $7 \times 6 + 4 = 30B$ $\therefore B = \frac{46}{30} = \frac{23}{15}$

Let $x = 0$: $4 = -6A$ $\therefore A = -\frac{4}{6} = -\frac{2}{3}$

So $\frac{7x+4}{5x^2-30x} \equiv \frac{23}{15(x-6)} - \frac{2}{15x}$

(b) $\frac{x+1}{3x^2-x-2} \equiv \frac{x+1}{(3x+2)(x-1)} \equiv \frac{A}{3x+2} + \frac{B}{x-1}$

***Multiply by** $(3x+2)(x-1)$:*

$$x + 1 \equiv A(x-1) + B(3x+2)$$

Equating coefficients of x: $1 = A + 3B$

Equating the constant terms: $1 = -A + 2B$

Solving simultaneously we get: $A = -\frac{1}{5}$ and $B = \frac{2}{5}$

So $\frac{x+1}{3x^2-x-2} \equiv \frac{2}{5(x-1)} - \frac{1}{5(3x+2)}$

Video: Partial fractions - what are they?
Video: Partial fractions - calculating constants

Solutions to Starter and E.g.s

Exercise
p101 5C Qu 1i, 2i, 3-11