Partial fractions with repeated factors

Starter

1. **(Review of last lesson)** Express $\dfrac{2x-3}{4x^2-11x+6}$ in partial fractions.

Working: $\dfrac{2x-3}{4x^2-11x+6} \equiv \dfrac{2x-3}{(4x-3)(x-2)} \equiv \dfrac{A}{4x-3} + \dfrac{B}{x-2}$

Multiply by $(4x-3)^2(x-2)$***:***

$2x-3 \equiv A(x-2) + B(4x-3)$

Equating coefficients of x: $2 = A + 4B$

Equating the constant terms: $-3 = -2A - 3B \quad \therefore 3 = 2A + 3B$

Solving simultaneously we get: $A = \dfrac{6}{5}$ and $B = \dfrac{1}{5}$

So $\dfrac{2x-3}{4x^2-11x+6} \equiv \dfrac{6}{5(4x-3)} + \dfrac{1}{5(x-2)}$

E.g. 1 Express the following as partial fractions.

(a) $\dfrac{8x+9}{(x-3)^2}$ (b) $\dfrac{2x-7}{(x+5)^2}$

Working: (a) $\dfrac{8x+9}{(x-3)^2} \equiv \dfrac{A}{x-3} + \dfrac{B}{(x-3)^2}$

Multiply by $(x-3)^2$***:***

$8x+9 \equiv A(x-3) + B$

Let $x = 3$: $8 \times 3 + 9 = B \quad \therefore B = 33$

Let $x = 0$: $9 = -3A + B \quad \therefore A = 8$

So $\dfrac{8x+9}{(x-3)^2} \equiv \dfrac{8}{x-3} + \dfrac{33}{(x-3)^2}$

(b) $\dfrac{2x-7}{(x+5)^2} \equiv \dfrac{A}{x+5} + \dfrac{B}{(x+5)^2}$

Multiply by $(x+5)^2$***:***

$2x-7 \equiv A(x+5) + B$

Equating coefficients of x: $2 = A$

Equating the constant terms: $-7 = 5A + B \quad \therefore B = -17$

So $\dfrac{2x-7}{(x+5)^2} \equiv \dfrac{2}{x+5} - \dfrac{17}{(x+5)^2}$

E.g. 2 Split into partial fractions

(a) $\dfrac{x}{(x+1)(x+2)^2}$ (b) $\dfrac{3x^2+6x+2}{(2x+3)(x+1)^2}$ (c) $\dfrac{x^2-7x-6}{x^2(x-3)}$

Working: (a) $\dfrac{x}{(x+1)(x+2)^2} \equiv \dfrac{A}{x+1} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$

Multiply by $(x+1)(x+2)^2$***:***

$$x \equiv A(x+2)^2 + B(x+1)(x+2) + C(x+1)$$

Let $x = -2$: $-2 = C(-2+1)$ $\therefore C = 2$

Let $x = -1$: $-1 = A(-1+2)^2$ $\therefore A = -1$

Let $x = 0$: $0 = 4A + 2B + C$ $\therefore B = 1$

So $\dfrac{x}{(x+1)(x+2)^2} \equiv \dfrac{1}{x+2} + \dfrac{2}{(x+2)^2} - \dfrac{1}{x+1}$

Alternatively using the equating coefficients method:

Equating coefficients of x^2: $0 = A + B$

Equating coefficients of x: $1 = 4A + 3B + C$

Equating the constant terms: $0 = 4A + 2B + C$

N.B. You can use your calculator to solve the equations.

Solving simultaneously we get: $A = -1, B = 1$ and $C = 2$

(b) $\dfrac{3x^2+6x+2}{(2x+3)(x+1)^2} \equiv \dfrac{A}{2x+3} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

Multiply by $(2x+3)(x+1)^2$***:***

$$3x^2+6x+2 \equiv A(x+1)^2 + B(2x+3)(x+1) + C(2x+3)$$

Let $x = -1$: $3 - 6 + 2 = C(-2+3)$ $\therefore C = -1$

Let $x = -\dfrac{3}{2}$: $\dfrac{27}{4} - 9 + 2 = A\left(-\dfrac{3}{2}+1\right)^2$ $\therefore A = -1$

Let $x = 0$: $2 = A + 3B + 3C$ $\therefore B = 2$

So $\dfrac{3x^2+6x+2}{(2x+3)(x+1)^2} \equiv \dfrac{2}{x+1} - \dfrac{1}{(x+1)^2} - \dfrac{1}{2x+3}$

(c) $\dfrac{x^2-7x-6}{x^2(x-3)} \equiv \dfrac{A}{x-3} + \dfrac{B}{x} + \dfrac{C}{x^2}$

Multiply by $x^2(x-3)$***:***

$$x^2 - 7x - 6 \equiv Ax^2 + Bx(x-3) + C(x-3)$$

Let $x = 3$: $9 - 21 - 6 = 9A$ $\therefore A = -2$

Let $x = 0$: $-6 = -3C$ $\therefore C = 2$

N.B. Since $x = 0$ has been used, choose another $x-$value.

Let $x = 1$: $1 - 7 - 6 = A - 2B - 2C$ $\therefore B = 3$

So $\dfrac{x^2-7x-6}{x^2(x-3)} \equiv \dfrac{3}{x} + \dfrac{2}{x^2} - \dfrac{2}{x-3}$

Video: Partial fractions - denominator contains repeated factors

Video (from 3:03): Solving simultaneous equations in 3 unknowns (Classwiz)

Partial fractions EQ

Solutions to Starter and E.g.s

Exercise

p103 5D Qu 1i, 2-8