

Product Rule

Starter

1. (Review of last lesson)

Differentiate these functions: (a) $f(x) = \cos^8(6x + 5)$ (b) $y = e^{\cos(4x-5)}$

Working: (a) $f(x) = \cos^8(6x + 5) = [\cos(6x + 5)]^8$
 $f'(x) = -48 \sin(6x + 5)\cos^7(6x + 5)$

(b) $\frac{dy}{dx} = -4 \sin(4x - 5)e^{\cos(4x-5)}$

2. Factorise $3x^2(3x - 2)^7 + 21x^3(3x - 2)^6$.

Working: $3x^2(3x - 2)^7 + 21x^3(3x - 2)^6 = 3x^2(3x - 2)^6[(3x - 2) + 7x]$
 $= 3x^2(10x - 2)(3x - 2)^6$

3. Show that $2x\sqrt{6x - 1} + \frac{3x^2}{\sqrt{6x - 1}}$ can factorise to $\frac{x(15x - 2)}{\sqrt{6x - 1}}$.

Working: $2x\sqrt{6x - 1} + \frac{3x^2}{\sqrt{6x - 1}} = \frac{2x(6x - 1)}{\sqrt{6x - 1}} + \frac{3x^2}{\sqrt{6x - 1}}$

N.B. $\sqrt{6x - 1} = \frac{6x - 1}{\sqrt{6x - 1}}$

$$2x\sqrt{6x - 1} + \frac{3x^2}{\sqrt{6x - 1}} = \frac{x}{\sqrt{6x - 1}} \left[2(6x - 1) + 3x \right]$$

$$= \frac{x(15x - 2)}{\sqrt{6x - 1}}$$

E.g. 1 Differentiate $y = x^2(3x + 1)^5$ with respect to x .

Working: I tend to use the poetry and go straight to line** below. However, many students find this structure helpful:

$$\begin{array}{l} u = x^2 \\ v = (3x + 1)^5 \end{array} \quad \begin{array}{c} \swarrow \quad \searrow \\ \nwarrow \quad \swarrow \end{array} \quad \begin{array}{l} u' = 2x \\ v' = 15(3x + 1)^4 \end{array}$$

$$\frac{dy}{dx} = 2x \times (3x + 1)^5 + 15(3x + 1)^4 \times x^2 \quad **$$

$$\frac{dy}{dx} = x(3x + 1)^4 [2(3x + 1) + 15x]$$

$$\frac{dy}{dx} = x(21x + 2)(3x + 1)^4$$

E.g. 2 Differentiate: (a) $f(x) = 3x^4 \cos x$. (b) $y = 6x\sqrt{4x-1}$

Working: (a) $u = 3x^4$ $u' = 12x^3$
 $v = \cos x$ $v' = -\sin x$

$$\frac{dy}{dx} = 12x^3 \times \cos x + (-\sin x) \times 3x^4$$

$$\frac{dy}{dx} = 3x^3(4 \cos x - x \sin x)$$

(b) $u = 6x$ $u' = 6$
 $v = (4x-1)^{\frac{1}{2}}$ $v' = 2(4x-1)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = 6 \times (4x-1)^{\frac{1}{2}} + 2(4x-1)^{-\frac{1}{2}} \times 6x$$

$$\frac{dy}{dx} = 6(4x-1)^{\frac{1}{2}} + 12x(4x-1)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = 6(4x-1)^{-\frac{1}{2}} \left[(4x-1) + 2x \right]$$

$$\frac{dy}{dx} = 6(4x-1)^{-\frac{1}{2}} \left[6x-1 \right]$$

$$\frac{dy}{dx} = \frac{6(6x-1)}{\sqrt{4x-1}}$$

E.g. 3 Find the stationary point(s) for the curve $y = xe^{x-x^2}$.

Working: $u = x$ $u' = 1$
 $v = e^{x-x^2}$ $v' = (1-2x)e^{x-x^2}$

$$\frac{dy}{dx} = 1 \times e^{x-x^2} + x(1-2x)e^{x-x^2}$$

$$\frac{dy}{dx} = e^{x-x^2} \left[1 + x(1-2x) \right]$$

$$\frac{dy}{dx} = e^{x-x^2} (1 + x - 2x^2)$$

A SP occurs when $\frac{dy}{dx} = 0$ so $e^{x-x^2} (1 + x - 2x^2) = 0$

Since $e^{x-x^2} > 0$, $1 + x - 2x^2 = 0$

$$x = 1 \text{ or } x = -\frac{1}{2}$$

Substitute the values into the original equation gives the stationary points as

$$(1, 1) \text{ and } \left(-\frac{1}{2}, -\frac{e^{\frac{3}{4}}}{2} \right)$$

Video: [Product rule](#)

[Solutions to Starter and E.g.s](#)

Exercise

p205 10B Qu 1i, 2i, 3-10, (FM – 11)