# Product Rule

**Starter**

1. **(Review of last lesson)**
Differentiate these functions: (a) $f(x) = \cos^8(6x+5)$ (b) $y = e^{\cos(4x-5)}$

***Working:*** (a) $f(x) = \cos^8(6x+5) = \left[\cos(6x+5)\right]^8$
$f'(x) = -48\sin(6x+5)\cos^7(6x+5)$

(b) $\dfrac{dy}{dx} = -4\sin(4x-5)e^{\cos(4x-5)}$

2. Factorise $3x^2(3x-2)^7 + 21x^3(3x-2)^6$.

***Working:*** 
$$\begin{aligned} 3x^2(3x-2)^7 + 21x^2(3x-2)^6 &= 3x^2(3x-2)^6[(3x-2)+7x] \\ &= 3x^2(10x-2)(3x-2)^6 \\ &= 6x^2(5x-1)(3x-2)^6 \end{aligned}$$

3. Show that $2x\sqrt{6x-1} + \dfrac{3x^2}{\sqrt{6x-1}}$ can factorise to $\dfrac{x(15x-2)}{\sqrt{6x-1}}$.

***Working:*** $2x\sqrt{6x-1} + \dfrac{3x^2}{\sqrt{6x-1}} = \dfrac{2x(6x-1)}{\sqrt{6x-1}} + \dfrac{3x^2}{\sqrt{6x-1}}$

***N.B.*** $\sqrt{6x-1} = \dfrac{6x-1}{\sqrt{6x-1}}$

$$\begin{aligned} 2x\sqrt{6x-1} + \frac{3x^2}{\sqrt{6x-1}} &= \frac{x}{\sqrt{6x-1}}\left[2(6x-1)+3x\right] \\ &= \frac{x(15x-2)}{\sqrt{6x-1}} \end{aligned}$$

***E.g. 1*** Differentiate $y = x^2(3x+1)^5$ with respect to $x$.

***Working:*** Using the poetry, you can go straight to line** below.
However, many students find this structure helpful:

$u = x^2$ $\quad$ $u' = 2x$
$v = (3x+1)^5$ $\quad$ $v' = 15(3x+1)^4$

$$\frac{dy}{dx} = 2x \times (3x+1)^5 + 15(3x+1)^4 \times x^2 \qquad **$$
$$\frac{dy}{dx} = x(3x+1)^4\left[2(3x+1)+15x\right]$$
$$\frac{dy}{dx} = = x(21x+2)(3x+1)^4$$

***E.g. 2*** Differentiate: (a) $f(x) = 3x^4 \cos x$. (b) $y = 6x\sqrt{4x - 1}$

***Working:*** (a)

$$u = 3x^4 \qquad u' = 12x^3$$
$$v = \cos x \qquad v' = -\sin x$$

$$\frac{dy}{dx} = 12x^3 \times \cos x + (-\sin x) \times 3x^4$$
$$\frac{dy}{dx} = 3x^3(4\cos x - x\sin x)$$

(b)

$$u = 6x \qquad u' = 6$$
$$v = (4x - 1)^{\frac{1}{2}} \qquad v' = 2(4x - 1)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = 6 \times (4x - 1)^{\frac{1}{2}} + 2(4x - 1)^{-\frac{1}{2}} \times 6x$$
$$\frac{dy}{dx} = 6(4x - 1)^{\frac{1}{2}} + 12x(4x - 1)^{-\frac{1}{2}}$$
$$\frac{dy}{dx} = 6(4x - 1)^{-\frac{1}{2}}\left[(4x - 1) + 2x\right]$$
$$\frac{dy}{dx} = 6(4x - 1)^{-\frac{1}{2}}\left[6x - 1\right]$$
$$\frac{dy}{dx} = \frac{6(6x - 1)}{\sqrt{4x - 1}}$$

***E.g. 3*** Find the stationary point(s) for the curve $y = xe^{x - x^2}$.

***Working:***

$$u = x \qquad u' = 1$$
$$v = e^{x - x^2} \qquad v' = (1 - 2x)e^{x - x^2}$$

$$\frac{dy}{dx} = 1 \times e^{x - x^2} + x(1 - 2x)e^{x - x^2}$$
$$\frac{dy}{dx} = e^{x - x^2}\left[1 + x(1 - 2x)\right]$$
$$\frac{dy}{dx} = e^{x - x^2}(1 + x - 2x^2)$$

A SP occurs when $\frac{dy}{dx} = 0$ so $e^{x - x^2}(1 + x - 2x^2) = 0$

Since $e^{x - x^2} > 0$, $\quad 1 + x - 2x^2 = 0$

$x = 1$ or $x = -\frac{1}{2}$

Substitute the values into the original equation gives the stationary points as

$(1, 1)$ and $\left(-\frac{1}{2}, -\frac{e^{-\frac{3}{4}}}{2}\right)$

Video: Product rule

Solutions to Starter and E.g.s

**Exercise**

p205 10B Qu 1i, 2i, 3-10, (FM — 11)