Radians

1. How many degrees are equivalent to one radian?

Hint: Arc length, $s = \frac{\theta}{360^o} \times 2\pi r$

Working: From GCSE, arc length: $s = \frac{\theta}{360^o} \times 2\pi r$

Since the angle is 1 radian: $s = r$ (i.e. arc length = radius)

$$r = \frac{\theta}{360^o} \times 2\pi r$$

Cancel r each side: $1 = \frac{2\pi\theta}{360^o}$

Rearranging gives: $\theta = \frac{360^o}{2\pi r} = \frac{180^o}{\pi r} \approx 57.3^o$

$1 \text{ radian} \approx 57.3^o$

E.g. 1 Convert 45^o to radians.

Working: $45^o \equiv 45 \times \frac{\pi}{180} = \frac{\pi}{4}$

Notice that since we have a fraction, we leave the angle in terms of π.

E.g. 2 Convert the following to degrees: (a) $\frac{2\pi}{3}$ (b) 1.85^c

Working: (a) $\frac{2\pi}{3} \equiv \frac{2\pi}{3} \times \frac{180}{\pi} = 120^o$

(b) $1.85^c \equiv 1.85^c \times \frac{180}{\pi} \approx 106^o$

Common angles

E.g. 3 Copy and complete this table.

Degress	0^o	30^o	45^o		90^o			150^o	180^o		360^o
Radians	0^c			$\frac{\pi}{3}$		$\frac{2\pi}{3}$	$\frac{3\pi}{4}$		π	$\frac{3\pi}{2}$	2π

Working:

Degress	0^o	30^o	45^o	60^o	90^o	120^o	135^o	150^o	180^o	270^o	360^o
Radians	0^c	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π

E.g. 4 Given that $\sin\frac{\pi}{6}=\frac{1}{2}$ state the values of the following without using a calculator:

(a) $\sin\frac{13\pi}{6}$ (b) $\sin\left(-\frac{\pi}{6}\right)$

(c) $\sin\frac{37\pi}{6}$ (d) $\sin\frac{11\pi}{6}$

Working:

(a) Periodic: $\sin\frac{13\pi}{6}=\sin\left(\frac{\pi}{6}+2\pi\right)=\sin\frac{\pi}{6}=\frac{1}{2}$

(b) Symmetry: $\sin\left(-\frac{\pi}{6}\right)=-\sin\left(\frac{\pi}{6}\right)=-\frac{1}{2}$

(c) Periodic: $\sin\frac{37\pi}{6}=\sin\left(\frac{\pi}{6}+6\pi\right)=\sin\frac{\pi}{6}=\frac{1}{2}$

(d) Combining: $\sin\frac{11\pi}{6}=\sin\left(2\pi-\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$

E.g. 5 Given that $\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$ state the values of the following without using a calculator:

(a) $\cos\frac{25\pi}{6}$ (b) $\cos\left(-\frac{\pi}{6}\right)$

(c) $\cos\left(-\frac{11\pi}{6}\right)$ (d) $\cos\frac{7\pi}{6}$

Working:

(a) $\cos\frac{25\pi}{6}=\cos\left(\frac{25\pi}{6}-4\pi\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

(b) $\cos\left(-\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

(c) $\cos\left(-\frac{11\pi}{6}\right)=\cos\left(-\frac{11\pi}{6}+2\pi\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

(d) $\cos\frac{7\pi}{6}=\cos\left(\frac{7\pi}{6}-2\pi\right)=\cos\left(-\frac{5\pi}{6}\right)$

$=-\cos\left(\pi-\frac{5\pi}{6}\right)=-\cos\frac{\pi}{6}=-\frac{\sqrt{3}}{2}$

Video: Radians

Solutions to Starter and E.g.s

Exercise

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