Range and Maximum Height

Starter

1. **(Review of last lesson)** A particle is projected from the ground with initial speed 8 m/s at an angle of 30^o above the horizontal.
   (a) Find the time it takes to reach its highest point.
   (b) Find its maximum height.
   (c) Find the time it takes to hit the ground again.
   (d) Find the horizontal displacement of the particle when it hits the ground again.

Working:

(a) At highest point, $v_y = 0$

$u_y = 8\sin 30,\ v_y = 0,\ a_y = -9.8,\ t = ?$

No s_y $\Rightarrow$ $v = u + at$

so $0 = 8\sin 30 - 9.8t$ so $t = 0.408$

(b) At maximum height, $v_y = 0$

$u_y = 8\sin 30,\ v_y = 0,\ a_y = -9.8,\ s_y = ?$

No t $\Rightarrow$ $v^2 = u^2 + 2as$

$0 = (8\sin 30)^2 + 2 \times -9.8 \times s_y$

$$s_y = \frac{(8\sin 30)^2}{2 \times 9.8} = \frac{40}{49} = 0.816 \text{ m}$$

(c) When it hits the ground, $s_y = 0$

$s_y = 0,\ u_y = 8\sin 30,\ a_y = -9.8,\ t = ?$

No v_y $\Rightarrow$ $s = ut + \frac{1}{2}at^2$

$$0 = 8t\sin 30 - \frac{1}{2} \times 9.8 \times t^2 \Rightarrow t(8\sin 30 - \frac{1}{2} \times 9.8 \times t) = 0$$

So $$t = \frac{8\sin 30}{4.9} = \frac{40}{49} = 0.816 \text{ s (3 s.f.)}$$

(d) $u_x = 8\cos 30,\ a_x = 0,\ t = \frac{40}{49},\ s_x = ?$

No v_x $\Rightarrow$ $s = ut + \frac{1}{2}at^2$

$s_x = 8\cos 30 \times 0.816 + 0 = 5.66$ m (3 s.f.)

E.g. 1 A projectile is launched from the ground with a speed of 15 m/s at an angle of 50° above the horizontal. Find:

(a) the time taken for the projectile to reach its maximum height

(b) the maximum height the projectile reaches above the ground.

Working:

(a) Maximum height is when $v_y = 0$

$u_y = 15\sin 50,\ a_y = -9.8,\ v_y = 0,\ t = ?$

No $s_y \Rightarrow \quad v = u + at$: $\quad 0 = 15\sin 50 - 9.8t$

$$t = \frac{15\sin 50}{9.8} = 1.17$$

The time taken to reach its maximum height is 1.17 s (3 s.f.)

(b) Maximum height is when $v_y = 0$

$u_y = 15\sin 50,\ a_y = -9.8,\ v_y = 0,\ s_y = ?$

No $t \quad \Rightarrow \quad v^2 = u^2 + 2as$: $\quad 0^2 = (15\sin 50)^2 - 2\times 9.8\times s_y$

$$s_y = \frac{(15\sin 50)^2}{2\times 9.8} = 6.74$$

The maximum height is 6.74 m

E.g. 2 A ball is thrown with an initial velocity of 30 m/s at an angle of 25°above the horizontal, from a height of 1.5 m above the ground.

(a) Calculate how long the ball is in the air before it hits the ground.

(b) Find the length of time the ball is at least 5 m above the ground.

(c) Calculate the particle's horizontal range.

Working:

(a) The ball hits the ground when $s_y = -1.5$.

$u_y = 30\sin 25,\ a_y = -9.8,\ s_y = -1.5,\ t = ?$

No $v_y \Rightarrow \quad s = ut + \frac{1}{2}at^2$: $\quad -1.5 = 30t\sin 25 + \frac{1}{2}\times(-9.8)t^2$

$4.9t^2 - 30t\sin 25 - 1.5 = 0$

Since $t > 0,\ t = 2.70$

The ball is in the air for 2.70 s (3 s.f.)

(b) 5 m above the ground $\Rightarrow s_y \geq 3.5$

$u_y = 30\sin 25,\ a_y = -9.8,\ t = ?$

No $v_y \Rightarrow \quad s = ut + \frac{1}{2}at^2$: $\quad 30t\sin 25 + \frac{1}{2}\times(-9.8)t^2 \geq 3.5$

$4.9t^2 - 30t\sin 25 + 3.5 \leq 0$

Solving $4.9t^2 - 30t\sin 25 + 3.5 = 0$ gives $t = 0.485$ & $t = 2.10$.

$t = 2.273\ldots - 0.314\ldots = 1.96$ s

Time spent above 5 m is 1.96 s

(c) The ball hits the ground when $t = 2.70$

$u_x = 30\cos 25,\ a_x = 0,\ t = 2.70,\ s_x = ?$

No $v_x \Rightarrow \quad s = ut + \frac{1}{2}at^2$: $\quad s_x = 30\cos 25\times 2.70 + 0 = 73.4$

The particle's horizontal range is 73.4 m

E.g. 3 A body is projected with velocity $(2\mathbf{i} + 11\mathbf{j})$ m/s from the ground. Find:

(a) the velocity of the body 0.8 s after projection

(b) the time of flight

(c) the maximum height.

Working:

(a) $\mathbf{u} = (2\mathbf{i} + 11\mathbf{j})$, $\mathbf{a} = -9.8\mathbf{j}$, $t = 0.8$, $\mathbf{v} = ?$

No $\mathbf{s}$ $\Rightarrow$ $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $\mathbf{v} = (2\mathbf{i} + 11\mathbf{j}) - 0.8 \times 9.8\mathbf{j}$

$\mathbf{v} = 2\mathbf{i} + 3.16\mathbf{j}$

The velocity of the body 0.8 s after projection is $(2\mathbf{i} + 3.16\mathbf{j})$ m/s

(b) The body returns to the ground when $s_y = 0$

$u_y = 11$, $a_y = -9.8$, $s_y = 0$, $t = ?$

No v $\Rightarrow$ $s = ut + \frac{1}{2}at^2$: $11t + \frac{1}{2} \times (-9.8) \times t^2 = 0$

Multiply by 2 and factorise: $t(22 - 9.8t) = 0$

$\therefore t = 0$ or $t = \frac{22}{9.8} = 2.24$

The time of flight is 2.24 s (3 s.f.)

(c) Maximum height $\Rightarrow$ $v_y = 0$

$u_y = 11$, $a_y = -9.8$, $v_y = 0$, $s_y = ?$

No t $\Rightarrow$ $v^2 = u^2 + 2as$: $0 = 11^2 - 2 \times 9.8 \times s_y$

$s_y = \frac{11^2}{2 \times 9.8} = 6.17$

The maximum height is 6.17 m (3 s.f.)

Video: Finding maximum height and range

Video: Finding the range given the maximum height

Video: Formula for maximum height

Video: Range of projectile

Solutions to Starter and E.g.s

Exercise

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Answers to additional questions

1. $\theta = 24.9^\circ$

2. 1.62 s

3. 4.35 m

4. (a) 5.27 m

   (b) Speed $= 18.0$ m/s; direction $= 80.5^\circ$ below the horizontal

5. (a) 1.56 s

   (b) 26.1 m

   (c) 17.9 m/s