

Related Rates of Change

Starter

1. **(Review of last lesson)** A curve has parametric equations $x = 3t + 6$ and $y = 2t - 8$. Find the area between the curve, the x -axis and the points where $t = -2$ and $t = 2$.

Working:
$$\int_{-2}^2 3(2t - 8)dt = -96 \text{ so area} = 96$$

2. Let $y = 3x^2$. The **rate of change** of y with respect to time, t , is 24. Use the **chain rule** to find the **rate of change** of x with respect to time when $x = 2$.

Working: "Find the **rate of change** of x with respect to time" means find $\frac{dx}{dt}$.

By the chain rule:
$$\frac{dx}{dt} = \frac{dy}{dt} \times \frac{dx}{dy} = \frac{\frac{dy}{dt}}{\frac{dy}{dx}}$$

$$\frac{dy}{dt} = 24$$

From $y = 3x^2 \Rightarrow \frac{dy}{dx} = 6x \Rightarrow \frac{dx}{dy} = \frac{1}{6x}$

$$\therefore \frac{dx}{dt} = \frac{dy}{dt} \times \frac{dx}{dy} = 24 \times \frac{1}{6x} = \frac{4}{x}$$

So when $x = 2$, $\frac{dx}{dt} = 2$.

3. A circle with area A is expanding at a rate of 5 cm^2 per second ($5 \text{ cm}^2 \text{ s}^{-1}$). Find the exact rate of increase of the radius, r , of the circle when the radius is 4 cm.

Working: "Find the exact rate of increase of the radius" means find $\frac{dr}{dt}$.

A circle is expanding at a rate of $5 \text{ cm}^2 \text{ s}^{-1} \Rightarrow \frac{dA}{dt} = 5$

N.B. Area because cm^2

By the chain rule:
$$\frac{dr}{dt} = \frac{dA}{dt} \times \frac{dr}{dA} = \frac{\frac{dA}{dt}}{\frac{dA}{dr}} \quad \text{so we need } \frac{dA}{dr}$$

For a circle $A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$

$$\therefore \frac{dr}{dt} = \frac{\frac{dA}{dt}}{\frac{dA}{dr}} = \frac{5}{2\pi r}$$

So when $r = 4$, $\frac{dr}{dt} = \frac{5}{8\pi} \text{ cm s}^{-1}$

E.g. 1 Air is pumped into a spherical ball of volume V which expands at a rate of $8 \text{ cm}^3 \text{ s}^{-1}$. Find the exact rate of increase of the radius, r , of the ball when the radius is 2 cm.

Working: “Find the exact rate of increase of the radius” means find $\frac{dr}{dt}$.

A spherical ball which expands at a rate of $8 \text{ cm}^3 \text{ s}^{-1} \Rightarrow \frac{dV}{dt} = 8$

By the chain rule: $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} = \frac{\frac{dV}{dt}}{\frac{dV}{dr}}$ *so we need $\frac{dV}{dr}$*

For a sphere $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$

$$\therefore \frac{dr}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dr}} = \frac{8}{4\pi r^2} = \frac{2}{\pi r^2}$$

So when $r = 2$, $\frac{dr}{dt} = \frac{2}{\pi \times 2^2} = \frac{1}{2\pi} \text{ cm s}^{-1}$

E.g. 2 A cuboid has dimensions 6, x and x . The volume of the cuboid, V , is increasing at a rate of $4 \text{ cm}^3 \text{ s}^{-1}$. Find the rate of increase of the side x when its length is 11 cm.

Working: “Find the rate of increase of the side x ” means find $\frac{dx}{dt}$.

“...volume of the cuboid is increasing at a rate of $4 \text{ cm}^3 \text{ s}^{-1}$ ” $\Rightarrow \frac{dV}{dt} = 4$

By the chain rule: $\frac{dx}{dt} = \frac{dV}{dt} \times \frac{dx}{dV} = \frac{\frac{dV}{dt}}{\frac{dV}{dx}}$ *so we need $\frac{dV}{dx}$*

For a cuboid, $V = 6x^2 \Rightarrow \frac{dV}{dx} = 12x$

$$\therefore \frac{dx}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dx}} = \frac{4}{12x} = \frac{1}{3x}$$

So when $x = 11$, $\frac{dx}{dt} = \frac{1}{3 \times 11} = \frac{1}{33} \text{ cm s}^{-1}$

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Exercise

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