Resolving Forces

Starter

1. **(Review of last lesson)** A javelin is thrown from a height of 1.5 m above the ground, at an angle of 38° to the horizontal. It reaches a maximum height of 12.5 m.
   (a) Calculate the javelin's initial velocity in m/s to 1 d.p.
   (b) Calculate the horizontal distance travelled by the javelin to the nearest metre.

Working: (a) $s_y = 12.5 - 1.5 = 11$

Maximum height $\Rightarrow$ $v_y = 0$

$u_y = u \sin 38,\ a_y = -9.8,\ s_y = 11,\ v_y = 0$

No t $\Rightarrow$ $v^2 = u^2 + 2as$: $0 = (u \sin 38)^2 - 2 \times 9.8 \times 11$

$$u^2 = \frac{2 \times 9.8 \times 11}{\sin^2 38} \approx 568.8 \quad \Rightarrow \quad u = 23.8$$

The javelin's initial velocity is 23.8 m/s (3 s.f.)

(b) Find time to hit ground $\Rightarrow$ $s_y = -1.5$

$u_y = 23.85 \sin 38,\ a_y = -9.8,\ s_y = -1.5,\ t = ?$

No t $\Rightarrow$ $s = ut + \frac{1}{2}at^2$: $-1.5 = 23.85t \sin 38 + \frac{1}{2} \times (-9.8) \times t^2$

$4.9t^2 - 23.85t \sin 38 - 1.5 = 0$

Since $t > 0$, $t = 3.10$ (3 s.f.)

Horizontal distance is s_x

$u_x = 23.85 \cos 38,\ a_x = 0,\ t = 3.0955,\ s_x = ?$

No v $\Rightarrow$ $s = ut + \frac{1}{2}at^2$: $s_x = 23.85 \cos 38 \times 3.0955 + 0$

$s_x = 58.18$

The horizontal distance travelled by the javelin is 58 m

2. A force of 12 N acts at an angle of 30° to the horizontal. How much of this force acts:
   (a) in the horizontal direction (b) in the vertical direction.

Working: (a) $12 \cos 30 = 12 \times \frac{\sqrt{3}}{2} = 6\sqrt{3} = 10.4$ N

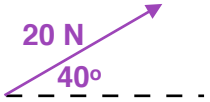
(b) $12 \sin 30 = 12 \times \frac{1}{2} = 6$ N

3. A force of R N acts at an angle of θ to the horizontal. How much of this force acts:
   (a) in the horizontal direction (b) in the vertical direction.

Working: (a) Horizontal $= R \cos \theta$

(b) Vertical $= R \sin \theta$

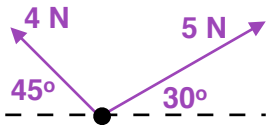
E.g. 1 Find the horizontal and vertical components of the force shown.



Working: $R(\rightarrow)$: Horizontal $= 20 \cos 40 = 15.3$ N,

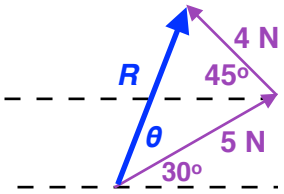
$R(\uparrow)$: Vertical $= 20 \sin 40 = 12.9$N

E.g. 2 Find the magnitude and direction of the resultant force acting on the particle.



Working: ***Triangle of forces method***

The **thick blue line** is the resultant force, R.



Using the cosine rule:

$R^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos 75$

$R = 5.54$

$\dfrac{\sin\theta}{4} = \dfrac{\sin 75}{5.536} \quad \Rightarrow \quad \theta = 44.3^\circ$

So the magnitude of the force is 5.54 N and the direction is $\theta = 74.3^\circ$

N.B. Remember, direction is measured in an anti-clockwise direction from the +ve $x-$axis.

Alternatively

Resolving in perpendicular directions method

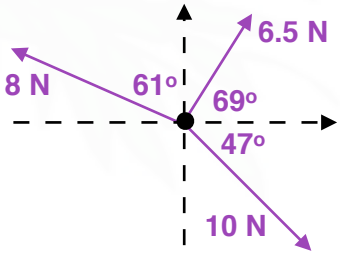
$R(\rightarrow)$: $\quad 5\cos 30 - 4\cos 45 \approx 1.502$

$R(\uparrow)$: $\quad 5\sin 30 + 4\sin 45 \approx 5.328$

By Pythagoras, the resultant, $R = \sqrt{1.502^2 + 5.328^2} = 5.54$ N

The angle is $\tan^{-1}\dfrac{5.328}{1.502} = 74.3^\circ$

E.g. 3 Find the magnitude and direction of the resultant force acting on the particle.



Working:

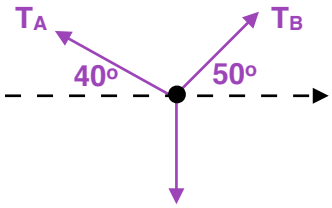
$R(\rightarrow)$: $\quad 6.5\cos 69 + 10\cos 47 - 8\sin 61 \approx 2.152$

$R(\uparrow)$: $\quad 8\cos 61 + 6.5\sin 69 - 10\sin 47 \approx 2.633$

By Pythagoras, the resultant, $R = \sqrt{2.152^2 + 2.633^2} = 3.40$ N (3 s.f.)

The angle is $\tan^{-1}\dfrac{2.633}{2.152} = 50.7^\circ$ (3 s.f.)

E.g. 4 An object of mass 7 kg is held in equilibrium by two strings, A and B, attached to a ceiling. String A is at 40^o to the horizontal and string B is at 50^o to the horizontal. Find the tension in the two strings.



Working:

$R(\rightarrow)$: $\quad T_A \cos 40 = T_B \cos 50$

$R(\uparrow)$: $\quad T_A \sin 40 + T_B \sin 50 = 7g$

Substitute $T_A = \dfrac{T_B \cos 50}{\cos 40}$ into $T_A \sin 40 + T_B \sin 50 = 7g$

$$\frac{T_B \cos 50}{\cos 40} \sin 40 + T_B \sin 50 = 7g$$

$$T_B \cos 50 \tan 40 + T_B \sin 50 = 7g \qquad \textit{since } \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$T_B(\cos 50 \tan 40 + \sin 50) = 7g \qquad \textit{factorise}$$

$$T_B = \frac{7g}{\cos 50 \tan 40 + \sin 50)} = 52.551$$

$$\therefore T_A = \frac{52.551 \cos 50}{\cos 40} = 44.1$$

The two tensions in the string are 44.1 N and 52.6 N

E.g. 5 A child pulls a toy car of mass 2 kg across a smooth floor with a force of 5 N at an angle of 35^o to the horizontal. Find the acceleration of the car.

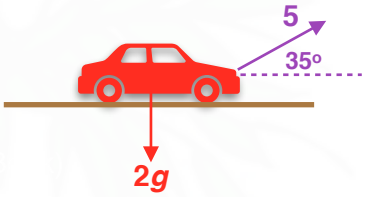
Working:

Smooth floor $\Rightarrow$ no friction

$F = ma(\rightarrow)$: $\quad 5\cos 35 = 2a$

$$a = \frac{5\cos 35}{2} = 2.04$$

The acceleration of the car is 2.04 m/s² (3 s.f.)



E.g. 6 A box of mass 15 kg is dragged along the floor at a constant speed of 1.2 m/s by means of a rope at 30^o to the horizontal. The tension in the rope is 50 N. Calculate:

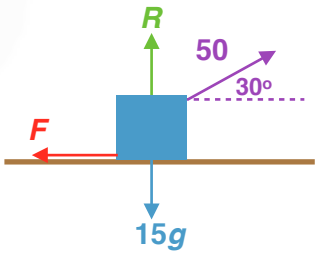
(a) the frictional force resisting the motion and

(b) the normal contact force from the floor.

Working:

(a) Let the friction force be F

Since the speed is constant we can $R(\rightarrow)$ or $F = ma(\rightarrow)$



$R(\rightarrow)$: $\quad F = 5\cos 30$

$$= 25\sqrt{3} = 43.4 \text{ N}$$

or

$F = ma(\rightarrow)$: $\quad F - 5\cos 30 = 0$ $\quad$ ***constant speed so*** $a = 0$

The frictional force resisting the motion is 43.4 N.

(b) $R(\uparrow)$: $\quad R + 50\sin 30 = 15g$

$$R = 15g - 50\sin 30$$

$$R = 122 \text{ N}$$

The normal contact force from the floor is 122 N

Video: Resolving a force
Video: Resultant forces (2 forces at an angle)

Resultant forces EQ

Solutions to Starter and E.g.s

Exercise
p472 21A Qu 1i, 2ii, 3-8