SUVAT in Vector Form

Starter

1. **(Review of AS material)** A car is moving along a straight road with uniform acceleration. The car passes a check-point A with a speed of 12 m/s and another check-point C with a speed of 32 m/s. The distance between A and C is 1100 m.
   (a) Find the time taken, in seconds, taken by the car to move from A to C.
   (b) Given that B is the midpoint of AC, find, in m/s to 1 dp, the speed with which the car passes B.

Working:

(a) $u = 12, v = 32, s = 1100, t = ?$

No a $\Rightarrow$ $s = \frac{1}{2}(u+v)t$: $1100 = \frac{1}{2}(12+32)t$

$\therefore t = 50$ s

(b) From the journey from A to C, acceleration can be found:

$u = 12, v = 32, s = 1100, a = ?$

No t $\Rightarrow$ $v^2 = u^2 + 2as$: $32^2 = 12^2 + 2a \times 1100$

$\therefore a = 0.4$

From A to B: $u = 12, s = 550, a = 0.4, v = ?$

No t $\Rightarrow$ $v^2 = u^2 + 2as$: $v^2 = 12^2 + 2 \times 0.4 \times 550$

$v^2 = 584$

$\therefore v = 24.2$ m/s (3 s.f.)

2. **(Review of AS material)** A particle A starts from rest at a point O and moves on a straight line with constant acceleration 2m/s^2. At the same instant another particle B, 12 m behind O, is moving with velocity 5m/s and has a constant acceleration of 3 m/s^2. How far from O are the particles when B overtakes A?

Working:

Let S be the distance from O when B overtake A and the let the time be T.

Particle A: $u = 0, a = 2, t = T, s = S$

Particle B: $u = 5, a = 3, t = T, s = S + 12$

No v $\Rightarrow$ $s = ut + \frac{1}{2}at^2$

Particle A: $S = \frac{1}{2} \times 2T^2$ $\Rightarrow$ $S = T^2$

Particle B: $S + 12 = 5T + \frac{1}{2} \times 3T^2$ $\Rightarrow$ $S + 12 = 5T + \frac{3}{2}T^2$

Replace S by T^2: $T^2 + 12 = 5T + \frac{3}{2}T^2$

Multiply by 2 and rearrange: $T^2 + 10T - 24 = 0$

$T = 2$ or $T = -12$ so $S = 4$

Distance from O is 4 m

E.g. 1 A particle, P, has initial velocity $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ ms^{-1} and constant acceleration $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$ ms^{-2}.

(a) Find the displacement and distance from the starting position after 4 seconds.

(b) Given that P started from the point with position vector $\begin{pmatrix} -6 \\ 7 \end{pmatrix}$, find its position after 4 seconds.

(c) Calculate the direction of motion after 5 seconds and calculate its speed at this time.

(d) Decide whether the the particle passes through the point $(0, 16)$.

Working:

(a) $\mathbf{u} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathbf{a} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, t = 4, \mathbf{s} = ?$

No $\mathbf{v}$ $\Rightarrow$ $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$: $\mathbf{s} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \times 4 + \frac{1}{2}\begin{pmatrix} 1 \\ 5 \end{pmatrix} \times 4^2$

Displacement, $\mathbf{s} = \begin{pmatrix} 16 \\ 36 \end{pmatrix}$ m

Distance, $|\mathbf{s}| = \sqrt{16^2 + 36^2} = 4\sqrt{97} \approx 39.4$ m

(b) To find its position, the displacement vector needs to be added to its starting position.

Position after 4 seconds $= \begin{pmatrix} -6 \\ 7 \end{pmatrix} + \begin{pmatrix} 16 \\ 36 \end{pmatrix} = \begin{pmatrix} 10 \\ 43 \end{pmatrix}$

(c) The direction of motion is equal to the velocity at that moment.

$\mathbf{u} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathbf{a} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, t = 4, \mathbf{v} = ?$

No $\mathbf{s}$ $\Rightarrow$ $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $\mathbf{v} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \end{pmatrix} \times 5$

Direction of motion, $\mathbf{v} = \begin{pmatrix} 7 \\ 24 \end{pmatrix}$

Speed, $|\mathbf{v}| = \sqrt{7^2 + 24^2} = 25$ ms^{-1}

(d) To pass through $(0, 16)$

Initial vector + displacement $= \begin{pmatrix} 0 \\ 16 \end{pmatrix}$

$$\begin{pmatrix} -6 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} \times t + \frac{1}{2}\begin{pmatrix} 1 \\ 5 \end{pmatrix} \times t^2 = \begin{pmatrix} 0 \\ 16 \end{pmatrix}$$

i: $-6 + 2t + \frac{1}{2}t^2 = 0$ $\quad t^2 + 4t - 12 = 0$

Solving for $t > 0$ gives $t = 2$

j: $7 - t + \frac{5}{2}t^2 = 16$ $\quad 5t^2 - 2t - 18 = 0$

Solving for $t > 0$ gives $t \approx 2.11$

Since the $t-$values are different P does not pass through $(0, 16)$

Video: SUVAT in 2-D
Video: SUVAT in 2-D Example

Exercise
p432 19B Qu 1i, 2-8