

## Set notation and Venn diagrams

### Starter

#### Key probability formulae

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A') = 1 - P(A)$$

If  $A$  and  $B$  are **mutually exclusive**:  $P(A \cap B) = 0 \Rightarrow P(A \cup B) = P(A) + P(B)$

If  $A$  and  $B$  are **independent**:  $P(A \cap B) = P(A) \times P(B)$

1. The events  $A$  and  $B$  are such that  $P(A) = 0.4$ ,  $P(B) = 0.45$  and  $P(A \cup B) = 0.68$ . Show that the events  $A$  and  $B$  are neither mutually exclusive nor independent.

**Working:**  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ :

$$0.68 = 0.4 + 0.45 - P(A \cap B)$$

$$P(A \cap B) = 0.17$$

Since  $P(A \cap B) \neq 0$ , events  $A$  and  $B$  are not mutually exclusive.

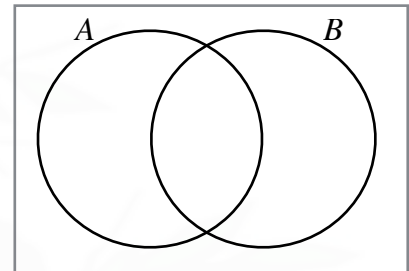
Since  $P(A) \times P(B) = 0.4 \times 0.45 = 0.18 \neq 0.17 = P(A \cap B)$ , events  $A$  and  $B$  are not independent.

2. **(Review of last lesson)** Draw three diagrams similar to the one on the right and shade the area given by:

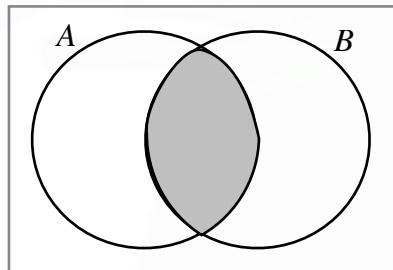
(a)  $A \cap B$  (i.e.  $A$  intersect  $B$ )

(b)  $A \cup B$  (i.e.  $A$  union  $B$ )

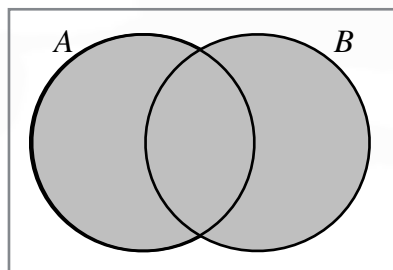
(c)  $A'$  (i.e.  $A$  complement)



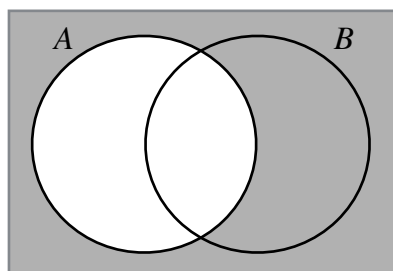
**Working:** (a)



(b)



(c)



**E.g. 1** Two events  $X$  and  $Y$  are such that  $P(X) = 0.7$ ,  $P(Y) = 0.4$  and  $P(X \cap Y) = 0.3$ . Find:

- (a)  $P(X \cup Y)$                       (b)  $P(X|Y)$                       (c)  $P(Y|X)$

**Working:** (a) 
$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

$$= 0.7 + 0.4 - 0.3$$

$$= 0.8$$

(b) 
$$P(X|Y) = \frac{P(Y \cap X)}{P(Y)} = \frac{0.3}{0.4} = \frac{3}{4}$$

(c) 
$$P(Y|X) = \frac{P(X \cap Y)}{P(X)} = \frac{0.3}{0.4 + 0.3} = \frac{3}{7}$$

**E.g. 2** Two events  $A$  and  $B$  are such that  $P(A) = 0.3$ ,  $P(B) = 0.5$  and  $P(A \cup B) = 0.55$ . Find:

- (a)  $P(A|B)$     (b)  $P(B|A)$

**Working:** (a) 
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.55 = 0.3 + 0.5 - P(A \cap B)$$

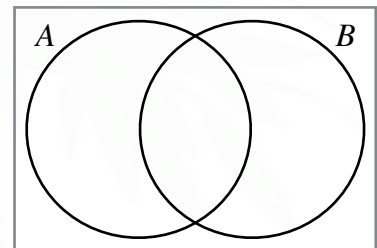
$$P(A \cap B) = 0.25$$

$$P(A|B) = \frac{P(B \cap A)}{P(B)} = \frac{0.25}{0.5} = \frac{1}{2}$$

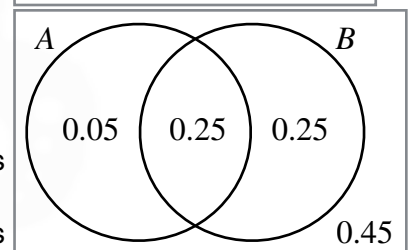
(b) 
$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.25}{0.3} = \frac{5}{6}$$

**E.g. 3** Two events  $A$  and  $B$  are such that  $P(A) = 0.3$ ,  $P(B) = 0.5$  and  $P(A \cup B) = 0.55$ .

- (a) Copy and complete the Venn diagram.  
 (b) Hence calculate:  
 (i)  $P(A|B')$   
 (ii)  $P(B'|A')$



**Working:** (a) Since  $P(A \cup B) = 0.55$ , 0.45 goes on the outside.  
 $P(A \cap B) = 0.25$  so 0.25 goes in the intersection.  
 Number in  $A$  but not in the intersection is  $0.3 - 0.25 = 0.05$ .  
 Number in  $B$  but not in the intersection is  $0.5 - 0.25 = 0.25$ .



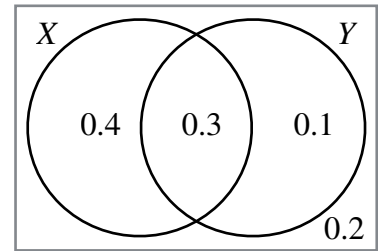
(b) (i) 
$$P(A|B') = \frac{P(B' \cap A)}{P(B')} = \frac{0.05}{0.05 + 0.45} = \frac{1}{10}$$

(ii) 
$$P(B'|A') = \frac{P(A' \cap B')}{P(A')} = \frac{0.45}{0.25 + 0.45} = \frac{9}{14}$$

**E.g. 4** Two events  $X$  and  $Y$  are such that  $P(X) = 0.7$ ,  $P(Y) = 0.4$  and  $P(X \cap Y) = 0.3$ . Find:

- (a) Draw a Venn diagram. (b)  $P(X|Y')$  (c)  $P(X'|Y)$

**Working:** (a)  $P(X \cup Y) = 0.8$   
 Outside =  $1 - 0.4 - 0.3 - 0.1 = 0.2$



(b) 
$$P(X|Y') = \frac{P(Y' \cap X)}{P(Y')}$$

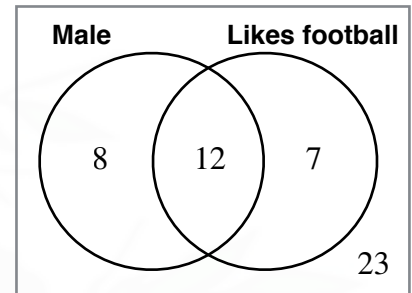
$$= \frac{0.4}{0.4} = \frac{2}{3}$$

(c) 
$$P(X'|Y) = \frac{P(Y \cap X')}{P(Y)}$$

$$= \frac{0.1}{0.3 + 0.1} = \frac{1}{4}$$

**E.g. 5** A person is picked at random. Find the probability that:

- (a) they are male given they like football  
 (b) they like football if they are female  
 (c) they don't like football given that they are female  
 (d) they are female given that they don't like football



**Working:** (a) "given they like football" means we only consider this part of the diagram so that the denominator is  $12 + 7$ .  
 The numerator is either 12 or 7.

Since we want the males, it must be 12  

$$P(\text{male} | \text{like football}) = \frac{12}{12 + 7} = \frac{12}{19}$$

(b) "if they are female" means we only consider females so that the denominator is  $7 + 23$ .

The numerator is either 7 or 23.  
 Since those females who like football, it must be 7  

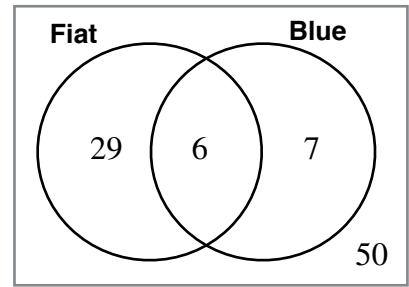
$$P(\text{like football} | \text{female}) = \frac{7}{7 + 23} = \frac{7}{30}$$

(c) 
$$P(\text{don't like football} | \text{female}) = \frac{23}{7 + 23} = \frac{23}{30}$$

(d) 
$$P(\text{female} | \text{don't like football}) = \frac{23}{8 + 23} = \frac{23}{31}$$

**E.g. 6** A car is picked at random. Find the probability that:

- (a) it is a fiat given that it is blue.
- (b) if it is not blue, then it is not a Fiat.
- (c) it is not blue given it is a Fiat.



**Working:** (a)  $P(\text{Fiat} | \text{blue}) = \frac{6}{6 + 7} = \frac{6}{13}$

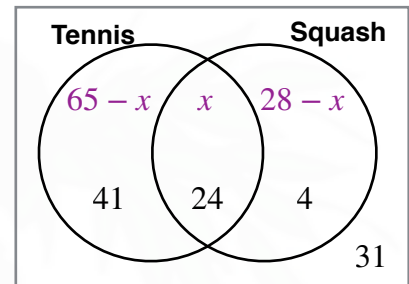
(b)  $P(\text{not Fiat} | \text{not blue}) = \frac{50}{29 + 50} = \frac{50}{79}$

(c)  $P(\text{not blue} | \text{Fiat}) = \frac{29}{29 + 6} = \frac{29}{35}$

**E.g. 7** At a sports club, members can play tennis and squash amongst other sport. 65 % of members play tennis, 28 % play squash and 31 % play neither.

- (a) Calculate what percentage of members play both tennis and squash and draw a Venn diagram and.
- (b) A member of the sports club is chosen at random. Find the probability that they:
  - (i) play tennis or squash but not both.
  - (ii) play squash given that they play tennis
  - (iii) play tennis given that they don't play squash.

**Working:** (a) 31 % play neither so 31 goes on the outside.  
Let  $x$  % play both tennis and squash.  
 $65 - x + 28 - x + x + 31 = 100$   
 $x = 24$



(b) (i)  $41\% + 4\% = 45\%$   
Probability = 0.45

(ii)  $P(S | T) = \frac{P(T \cap S)}{P(T)} = \frac{24}{41 + 24} = \frac{24}{65}$

(iii)  $P(T | S') = \frac{P(S' \cap T)}{P(S')} = \frac{41}{41 + 31} = \frac{41}{72}$

**Video:** [Venn diagrams](#)

**Video:** [Three set problems](#)

**Video:** [Notation and defining regions](#)

**Video:** [Probability in Venn diagrams](#)

**Video:** [Conditional probability in Venn diagrams](#)

**Video:** [P\(A∪B\) and Mutually Exclusive Events](#)

**Exam questions:** [Venn diagrams](#)

[Solutions to Starter and E.g.s](#)

**Exercise**

p360 16A Qu 1ace..., 2i, 3i, 4ace..., 5i, 6-19