Simplifying rational expression

Starter

1. (Review of last lesson) The polynomial $px^3 + 5x^2 + qx + 8$ has factors $3x - 1$ and $x + 4$. Find the values of p and q.

Working: Let $f(x) = px^3 + 5x^2 + qx + 8$.

$3x - 1$ is a factor $\Rightarrow f\left(\frac{1}{3}\right) = 0$: $\frac{p}{27} + \frac{5}{9} + \frac{q}{3} + 8 = 0$

Multiply by 27 and rearrange: $p + 9q = -231$

$x + 4$ is a factor $\Rightarrow f(-4) = 0$: $-64p + 80 - 4q + 8 = 0$

Divide by 4 and rearrange: $16p + q = 22$

Solving simultaneously gives: $p = 3$ and $q = -26$

2. (Review of GCSE material)

Simplify: (a) $\frac{x^2 + 5x - 6}{x^2 - 4x + 3}$ (b) $\frac{x^2 - 4}{x + 3} \times \frac{3}{x - 2}$

Working: (a) $\frac{x^2 + 5x - 6}{x^2 - 4x + 3} = \frac{(x+6)(x-1)}{(x-1)(x-3)} = \frac{(x+6)}{(x-3)}$

(b) $\frac{x^2 - 4}{x + 3} \times \frac{3}{x - 2} = \frac{(x+2)(x-2)}{x+3} \times \frac{3}{x-2} = \frac{3(x+2)}{x+3}$

E.g. 1 Simplify: (a) $\frac{x+2}{2x+3} \div \frac{2x+4}{8x+12}$ (b) $\frac{5x-1}{2x^2+x-3} \div \frac{1}{2x^2+7x+6}$

Working: (a)

$$\frac{x+2}{2x+3} \div \frac{2x+4}{8x+12} = \frac{x+2}{2x+3} \times \frac{8x+12}{2x+4}$$
$$= \frac{x+2}{2x+3} \times \frac{4(2x+3)}{2(x+2)}$$
$$= \frac{4}{2} = 2$$

(b)

$$\frac{5x-1}{2x^2+x-3} \div \frac{1}{2x^2+7x+6} = \frac{5x-1}{2x^2+x-3} \times \frac{2x^2+7x+6}{1}$$
$$= \frac{5x-1}{(2x+3)(x-1)} \times \frac{(2x+3)(x+2)}{1}$$
$$= \frac{5x-1}{x-1} \times \frac{x+2}{1}$$
$$= \frac{(5x-1)(x+2)}{x-1}$$

E.g. 2 Without using polynomial division, find the quotient and remainder when:

(a) $2x^2 + 9x - 4$ is divided by $x - 2$

(b) $x^3 + 4x^2 - 7$ is divided by $x^2 - 3$

(c) $3x^3 - 5$ is divided by $x + 4$

Working: (a)

$$\frac{2x^2 + 9x - 4}{x - 2} \equiv Ax + B + \frac{C}{x + 1}$$

$$2x^2 + 9x - 4 \equiv (x - 2)(Ax + B) + C$$

Equating coefficient: x^2: $2 = A$

x: $9 = -2A + B$ $\quad \therefore B = 13$

constant: $-4 = -2B + C$ $\quad \therefore C = 22$

$$\frac{2x^2 + 9x - 4}{x - 2} \equiv 2x + 13 + \frac{22}{x + 1}$$

The quotient is $2x + 13$ and the remainder is 22.

(b) Deg. of quotient = Deg. of dividend − Deg. of divisor $= 3 - 2 = 1$

Quotient is of the form $Ax + B$

Degree of remainder = Degree of divisor $-1 = 2 - 1 = 1$

Remainder is of the form $Cx + D$

$$\frac{x^3 + 4x^2 - 7}{x^2 - 3} \equiv Ax + B + \frac{Cx + D}{x^2 - 3}$$

$$x^3 + 4x^2 - 7 \equiv (x^2 - 3)(Ax + B) + Cx + D$$

Equating coefficient: x^3: $1 = A$

x^2: $4 = B$

x: $0 = -3A + C$ $\quad \therefore C = 3$

constant: $-7 = -3B + D$ $\quad \therefore D = 5$

$$\frac{x^3 + 4x^2 - 7}{x^2 - 3} \equiv x + 4 + \frac{3x + 5}{x^2 - 3}$$

The quotient is $x + 4$ and the remainder is $3x + 5$.

(c) Deg. of quotient = Deg. of dividend − Deg. of divisor $= 3 - 1 = 2$

Quotient is of the form $Ax^2 + Bx + C$

Degree of remainder = Degree of divisor $-1 = 1 - 1 = 0$

Remainder is a constant i.e. D

$$\frac{3x^3 - 5}{x + 4} \equiv Ax^2 + Bx + C + \frac{D}{x + 4}$$

$$3x^3 - 5 \equiv (x + 4)(Ax^2 + Bx + C) + D$$

Equating coefficient: x^3: $3 = A$

x^2: $0 = 4A + B$ $\quad \therefore B = -12$

x: $0 = 4B + C$ $\quad \therefore C = 48$

constant: $-5 = 4C + D$ $\quad \therefore D = -197$

$$\frac{3x^3 - 5}{x + 4} \equiv 3x^2 - 12x + 48 - \frac{197}{x + 4}$$

The quotient is $3x^2 - 12x + 48 = 3(x^2 - 4x + 12)$ and the remainder is -197.

Video: Simplifying algebraic fractions

Video: Multiplication of algebraic fractions

Simplifying algebraic fractions EQ

Algebraic long division EQ

Solutions to Starter and E.g.s

Exercise
p97 5B Qu 1i, 2i, 3i, 4i, 5-13, (14-16 red)