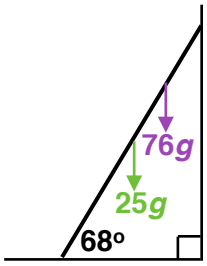
Supported Beams

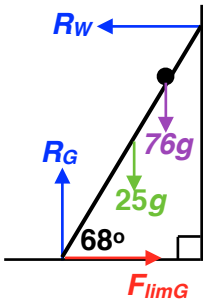
Starter

1. **(Review of last lesson)**

A uniform ladder of mass 25 kg and length 9 m rest with one end on rough, horizontal ground and the other end against a smooth, vertical wall. the ladder makes an angle of 68^o with the horizontal, as shown. A window cleaner of mass 76 kg stands two-thirds of the way up the ladder, and the ladder is in equilibrium. Find the range of possible values of the coefficient of friction between the ground and the ladder.



Working: $F_{limG} = \mu R_G$ so we need to find F_{limG} and R_G.



$\circlearrowleft$ about R_G:

$$25g\cos 68 \times 4.5 + 76g\cos 68 \times 6 = R_W \sin 68 \times 9$$

$$\frac{25g\cos 68 \times 4.5 + 76g\cos 68 \times 6}{\sin 68 \times 9} = R_W$$

$$R_W = 250.1...$$

$R(\rightarrow)$: $F_{limG} = R_W = 250.1...$N

$R(\uparrow)$: $R_G = 25g + 76g = 101g$ N

$F_{lim} = \mu R$: $\mu = \dfrac{F_{limG}}{R_G}$

$$\mu = \frac{250.1...}{101g}$$

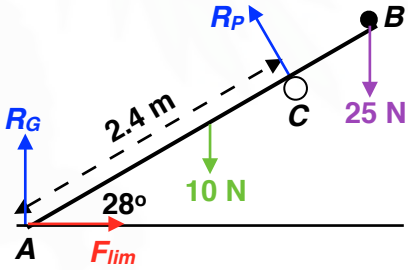
$$\mu \approx 0.253$$

The coefficient of friction must be greater than or equal to 0.253 (3 s.f.).

E.g. 1 A uniform rod, AB, of length 3.3 m and weight and 10 N, rests with A on rough horizontal ground.

The rod is supported by a smooth peg at C, where $AC = 2.4$ m, in such a way that the rod makes an angle of 28^o with the ground.

A particle of weight 25 N is placed at B.



Given that the rod is in limiting equilibrium, find

(a) the magnitude of the normal reaction, R_P, at the peg and

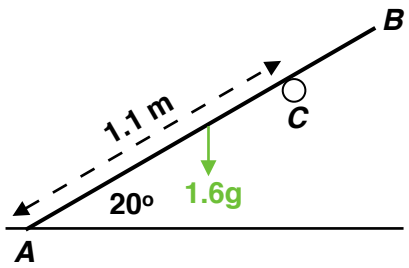
(b) the frictional force, F_{lim}, between the rod and the ground.

Working: (a) $\circlearrowleft$ about A: $2.4R_P = (25\cos 28 \times 3.3) + (10\cos 28 \times 1.65)$

$$R_P = 36.4 \text{ N (3 s.f.)}$$

(b) $R(\rightarrow)$: $F_{lim} = R_P \sin 28 = 17.1$ N (3 s.f.)

E.g. 2 A uniform beam, AB, of mass 1.6 kg and the length 1.5 m, rests with A on rough, horizontal ground. The beam is supported by a smooth peg at C, where $AC = 1.1$ m, so that it makes an angle of 20^o with the horizontal, as shown.



The beam is on the point of slipping.

Find:

(a) the magnitude of the normal reaction of the peg on the beam,
(b) the magnitude of the normal reaction of the ground on the beam,
(c) the magnitude of the fictional force between the ground and the beam,
(d) the coefficient of friction between the ground and the beam.

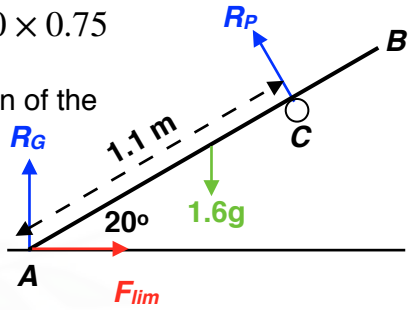
Hint: draw a diagram with all the forces on.

Working:

(a) $\circlearrowleft$ about A: $1.1R_P = 1.6g \cos 20 \times 0.75$

$R_P \approx 10.0$ N

The magnitude of the normal reaction of the peg on the beam is 10.0 N (3 s.f.)



(b) $R(\uparrow)$:

$R_G + R_P \cos 20 = 1.6g$

$R_G = 1.6g - 10.0... \cos 20$

$R_G \approx 6.24$ N

The magnitude of the normal reaction of the ground on the beam is 6.24 N (3 s.f.)

(c) $R(\rightarrow)$: $F_{lim} = R_P \sin 20$

$F_{lim} = 10.0... \sin 20$

$F_{lim} \approx 3.44$ N

The magnitude of the fictional force between the ground and the beam is 3.44 N (3 s.f.)

(d) $F_{lim} = \mu R$: $\mu = \dfrac{F_{limG}}{R_G}$

$\mu = \dfrac{3.44...}{6.24...}$

$\mu \approx 0.551$

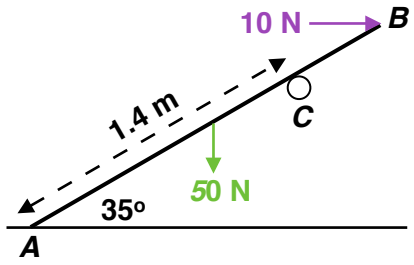
The coefficient of friction between the ground and the beam is 0.551 (3 s.f.)

N.B. $10.0...$ indicates that the calculator value is used in the calculation.

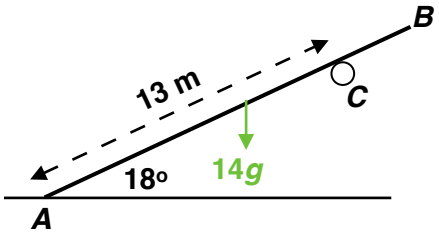
Solutions to Starter and E.g.s

Exercise

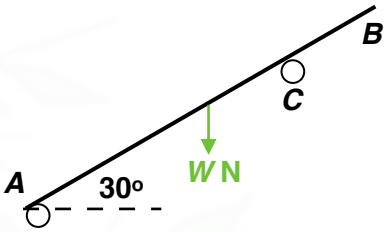
1. A uniform beam, AB, of weight 50 N and length 2 m, rests with A on rough, horizontal ground, as shown. The beam is supported at an angle of 35^o to the ground by a smooth peg at C, where $AC = 1.4$ m. A horizontal force of magnitude 10 N is applied to the beam at B. Find the range of values of the coefficient of friction between the ground and the beam for which the beam will remain in equilibrium.



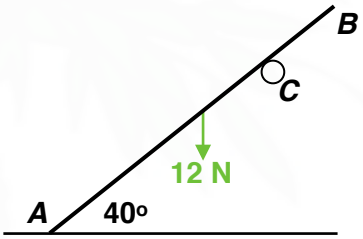
2. A uniform beam, AC, of mass 14 kg and length 15 m, rests with A on rough, horizontal ground. The beam is supported by a smooth peg at C, where $AC = 13$ m, so that it makes an angle of 18^o with the horizontal, as shown. The beam is on the point of slipping. Find the coefficient of friction between the ground and the beam at A.



3. A uniform beam, AB, of weight W N rests in limiting equilibrium at an angle of 30^o to the horizontal on a rough peg at A and a smooth peg at C, where $AC = 0.75AB$. The reaction forces at A and C are both perpendicular to the beam. Find the coefficient of friction between the peg and the beam at A.



4. A uniform beam AB of weight 12 N and the length 5 m rests on a smooth surface at A and a rough peg at C, 4 m from A. The beam makes an angle of 40^o with the horizontal, as shown. Given that the beam is in equilibrium, find the range of possible values of the coefficient of friction between the beam and the peg.



Answers to exercise

1. $\mu \geq 0.59$
2. $\mu = 0.35$
3. $\mu = 1.73$
4. $\mu \geq 0.84$