

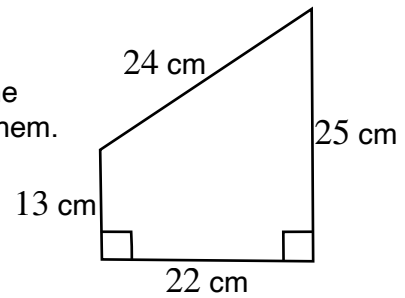
Trapezium Rule

Starter

1. (Review of last lesson) Find the area of this shape.

Working: Area of trapezium equals half the sum of the parallel sides times the distance between them.

$$\begin{aligned} \text{Area of trapezium} &= \frac{1}{2}(a + b)h \\ &= \frac{1}{2} \times (13 + 25) \times 22 \\ &= 38 \times 11 \\ &= 418 \end{aligned}$$



- E.g. 1** Use the trapezium rule with 4 strips (i.e. trapezia) to approximate $\int_0^{0.8} \cos x dx$, giving your answer to 3 s.f.

N.B. Make sure your calculator is in **radians**.

Working: Trapezia width = $\frac{0.8 - 0}{4} = 0.2$

x -values: start at $x = 0$ since this is the lower limit of the integral.
Add 0.2 each time to the x -value because this is the trapezia width
It can be useful to form a table to write down the values

x	0	0.2	0.4	0.6	0.8
y	$\cos 0$	$\cos 0.2$	$\cos 0.4$	$\cos 0.6$	$\cos 0.8$

$$\begin{aligned} \text{Area} &\approx \frac{1}{2} \times 0.2 \times (\cos 0 + 2(\cos 0.2 + \cos 0.4 + \cos 0.6) + \cos 0.8) \\ &= 0.715 \text{ (3 s.f.)} \end{aligned}$$

- E.g. 2** Use the trapezium rule with 3 strips (i.e. trapezia) to approximate $\int_1^7 \ln x dx$, giving your answer exactly in the form $\ln a$ where a is an integer.

Working: Trapezia width = $\frac{7 - 1}{3} = 2$

x -values: start at $x = 1$ since this is the lower limit of the integral.
Add 2 each time to the x -value because this is the trapezia width

x	1	3	5	7
y	0	$\ln 3$	$\ln 5$	$\ln 7$

$$\begin{aligned} \text{Area} &\approx \frac{1}{2} \times 2 \times (0 + 2(\ln 3 + \ln 5) + \ln 7) \\ &= 2 \ln 15 + \ln 7 \\ &= \ln 225 + \ln 7 && \text{because } 15^2 = 225 \text{ (3rd law of logs)} \\ &= \ln 1575 && \text{because } 225 \times 7 = 1575 \text{ (1st law of logs)} \end{aligned}$$

E.g. 3 Find an approximation for $\int_0^{0.6} x^2 dx$ using 3 strips with the trapezium rule and find the percentage error for the approximation given that the actual value is 0.072.

Working: Trapezia width = $\frac{0.6 - 0}{3} = 0.2$

x -values: start at $x = 0$ since this is the lower limit of the integral.
Add 0.2 each time to the x -value because this is the trapezia width

x	0	0.2	0.4	0.6
y	0^2	0.2^2	0.4^2	0.6^2

$$\text{Area} \approx \frac{1}{2} \times 0.2 \times (0^2 + 2(0.2^2 + 0.4^2) + 0.6^2)$$

$$= 0.076$$

$$\text{Percentage error} = \frac{|\text{real value} - \text{estimate}|}{\text{real value}} \times 100\%$$

$$\text{Percentage error} = \frac{|0.072 - 0.076|}{0.072} \times 100\%$$

$$= 5.56\% \text{ (3 s.f.)}$$

E.g. 4 Use the trapezium rule with 4 intervals to evaluate $\int_0^4 e^x dx$ to 4 s.f. Given that the correct value is $e^4 - 1$, find the percentage error of your value to 1 d.p..

Working: Trapezia width = $\frac{4 - 0}{4} = 1$

x -values: start at $x = 0$ since this is the lower limit of the integral.
Add 1 each time to the x -value because this is the trapezia width

x	0	1	2	3	4
y	e^0	e^1	e^2	e^3	e^4

$$\text{Area} \approx \frac{1}{2} \times 1 \times (e^0 + 2(e^1 + e^2 + e^3) + e^4)$$

$$= 57.99$$

$$\text{Percentage error} = \frac{|\text{real value} - \text{estimate}|}{\text{real value}} \times 100\%$$

$$\text{Percentage error} = \frac{|(e^4 - 1) - 57.99|}{e^4 - 1} \times 100\%$$

$$= 8.2\% \text{ (3 s.f.)}$$

Video: [Trapezium rule \(including under-and over-estimates\)](#)

[Trapezium rule EQ](#)

[Solutions to Starter and E.g.s](#)

Exercise

p339 15B Qu 1aci, 3ace, 4-7