Trapezium Rule

E.g. 1 Use the trapezium rule with 4 strips (i.e. trapezia) to approximate $\int_0^{0.8} \cos x \, dx$, giving your answer to 3 s.f.

N.B. Make sure your calculator is in ***radians***.

Working: Trapezia width $= \dfrac{0.8 - 0}{4} = 0.2$

$x-$values: start at $x = 0$ since this is the lower limit of the integral.
Add 0.2 each time to the $x-$value because this is the trapezia width
It can be useful to form a table to write down the values

x	0	0.2	0.4	0.6	0.8
y	cos 0	cos 0.2	cos 0.4	cos 0.6	cos 0.8

$$\text{Area} \approx \frac{1}{2} \times 0.2 \times \Big(\cos 0 + 2(\cos 0.2 + \cos 0.4 + \cos 0.6) + \cos 0.8\Big)$$
$$= 0.615 \text{ (3 s.f.)}$$

E.g. 2 Use the trapezium rule with 3 strips (i.e. trapezia) to approximate $\int_1^7 \ln x \, dx$, giving your answer exactly in the form $\ln a$ where a is an integer.

Working: Trapezia width $= \dfrac{7 - 1}{3} = 2$

$x-$values: start at $x = 1$ since this is the lower limit of the integral.
Add 2 each time to the $x-$value because this is the trapezia width

x	1	3	5	7
y	0	ln 3	ln 5	ln 7

$$\text{Area} \approx \frac{1}{2} \times 2 \times \Big(0 + 2(\ln 3 + \ln 5) + \ln 7\Big)$$
$$= 2\ln 15 + \ln 7$$
$$= \ln 225 + \ln 7 \qquad \textit{because } 15^2 = 225 \textit{ (3rd law of logs)}$$
$$= \ln 1575 \qquad \textit{because } 225 \times 7 = 1575 \textit{ (1st law of logs)}$$

E.g. 3 Find an approximation for $\int_0^{0.6} x^2 dx$ using 3 strips with the trapezium rule and find the percentage error for the approximation given that the actual value is 0.072.

Working: Trapezia width $= \dfrac{0.6-0}{3} = 0.2$

$x-$values: start at $x = 0$ since this is the lower limit of the integral.
Add 0.2 each time to the $x-$value because this is the trapezia width

x	0	0.2	0.4	0.6
y	0^2	0.2^2	0.4^2	0.6^2

$$\text{Area} \approx \frac{1}{2} \times 0.2 \times \left(0^2 + 2(0.2^2 + 0.4^2) + 0.6^2\right)$$
$$= 0.076$$

$$\text{Percentage error} = \frac{\left|\text{real value} - \text{estimate}\right|}{\text{real value}} \times 100\,\%$$

$$\text{Percentage error} = \frac{\left|0.072 - 0.076\right|}{0.072} \times 100\,\%$$
$$= 5.56\,\% \text{ (3 s.f.)}$$

E.g. 4 Use the trapezium rule with 4 intervals to evaluate $\int_0^4 e^x dx$ to 4 s.f. Given that the correct value is $e^4 - 1$, find the percentage error of your value to 1 d.p..

Working: Trapezia width $= \dfrac{4-0}{4} = 1$

$x-$values: start at $x = 0$ since this is the lower limit of the integral.
Add 1 each time to the $x-$value because this is the trapezia width

x	0	1	2	3	4
y	e^0	e^1	e^2	e^3	e^4

$$\text{Area} \approx \frac{1}{2} \times 1 \times \left(e^0 + 2(e^1 + e^2 + e^3) + e^4\right)$$
$$= 57.99$$

$$\text{Percentage error} = \frac{\left|\text{real value} - \text{estimate}\right|}{\text{real value}} \times 100\,\%$$

$$\text{Percentage error} = \frac{\left|(e^4 - 1) - 57.99\right|}{e^4 - 1} \times 100\,\%$$
$$= 8.2\,\% \text{ (3 s.f.)}$$

Video: Trapezium rule (including under-and over-estimates)

Trapezium rule EQ

Solutions to Starter and E.g.s

Exercise
p339 15B Qu 1aci, 3ace, 4-7