

## Triangles and circles

### Starter

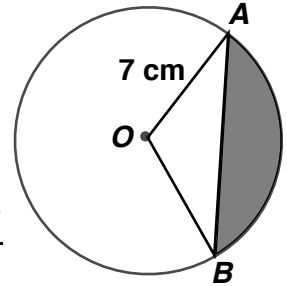
1. Write down the formula for the area of a segment when the angle is given in radians.

**Working:** Area of segment = Area of Sector – Area of Triangle

$$\begin{aligned} \text{Area of segment} &= \frac{1}{2}r^2\theta^c - \frac{1}{2}r^2 \sin \theta^c \\ &= \frac{1}{2}r^2(\theta^c - \sin \theta^c) \end{aligned}$$

2. Find the value of the shaded area given that  $\angle AOB = \frac{2\pi}{3}$ .

Give your answer exactly.



**Working:**

$$\begin{aligned} \text{Area of segment} &= \frac{1}{2}r^2\theta^c - \frac{1}{2}r^2 \sin \theta^c \\ \text{Area of segment} &= \frac{1}{2} \times 7^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 7^2 \times \sin \frac{2\pi}{3} \\ &= \frac{49}{2} \left( \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) \end{aligned}$$

The value of the shaded area is  $\frac{49}{2} \left( \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right)$  or  $\frac{49}{12} (4\pi - 3\sqrt{3})$ .

- E.g. 1** Find the area of the segment cut off by a chord of length 10 cm from a circle of radius 13 cm.

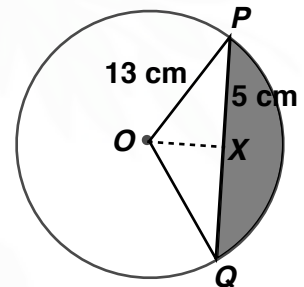
**Working:** *By drawing a dotted line, two right-angled triangles are created.*

$$\sin P\hat{O}X = \frac{5}{13} \Rightarrow P\hat{O}X \approx 0.395^c$$

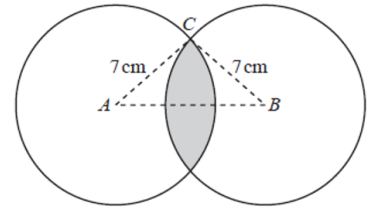
$$\text{So } P\hat{O}Q \approx 2 \times 0.395^c \approx 0.7896^c$$

$$\begin{aligned} \text{Area of segment} &= \frac{1}{2}r^2\theta^c - \frac{1}{2}r^2 \sin \theta^c \\ &= \frac{1}{2} \times 13^2 (0.7896^c - \sin 0.7896^c) \\ &\approx 6.72 \end{aligned}$$

The area of the segment is 6.72 cm<sup>2</sup> (3 s.f.).



**E.g. 2** The diagram shows two circles of radius 7 cm with centres  $A$  and  $B$ . The distance  $AB$  is 12 cm and the point  $C$  lies on both circles. The region common to both circles is shaded.

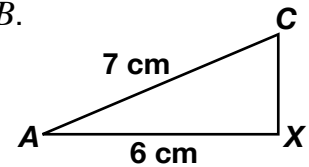


- (a) Find the angle  $CAB$  in radians to 4 s.f..  
 (b) Find the area of the shaded region to 3 s.f..

**Working:** (a) Let  $X$  be the point half-way between  $A$  and  $B$ .

$$\cos \hat{CAB} = \cos \hat{CAX} = \frac{6}{7}$$

$$\angle CAB = 0.5411^c$$



- (b)  $2 \times \angle CAB = 1.0822^c$

$$\text{Half the shaded area} = \frac{1}{2}r^2\theta^c - \frac{1}{2}r^2 \sin \theta^c$$

$$= \frac{1}{2} \times 7^2 (1.0822^c - \sin 1.0822^c)$$

$$\approx 4.8806$$

The area of the shaded region is  $9.76 \text{ cm}^2$  (3 s.f.).

**Video:** [Arcs, sectors and segments](#)

[Arcs, sectors and segments EQ](#)

[Solutions to Starter and E.g.s](#)

**Exercise**

p154 7E Qu 1-4, (5 red)