Using Z—scores and the standard distribution

Starter

1. For $X \sim \text{N}(165, 8^2)$, use your calculator to find:
   (a) $P(X < 160)$ (b) $P(X \geq 157)$
   (c) $P(X \leq 162, X > 170)$ (d) $P(|X - 165| < 14)$
   Give your answers to 4 d.p..

Working: (a) $P(X < 160) = 0.2656$ (4 d.p.)

(b) $P(X \geq 157) = 0.8413$ (4 d.p.)

(c) $P(X \leq 162, X > 170) = 1 - P(162 < X \leq 170)$
$= 1 - 0.38018$
$= 0.6198$ (4 d.p.)

...or...

$P(X \leq 162, X > 170) = P(X \leq 162) + P(X > 170)$
$= 0.353830 + 0.265986$ (6 d.p.)
$= 0.6198$ (4 d.p.)

(c) $P(|X - 165| < 14) = P(165 - 14 < X < 165 + 14)$
$= P(151 < X < 179)$
$= 0.9199$ (4 d.p.)

E.g. 1 Find the corresponding Z—value to the given value of X:
(a) $X \sim \text{N}(18, 4), x = 21$ (b) $X \sim \text{N}(256, 9^2), x = 238$

Working: (a) $z = \dfrac{x - \mu}{\sigma}$: $z = \dfrac{21 - 18}{4} = 0.75$

(b) $z = \dfrac{x - \mu}{\sigma}$: $z = \dfrac{238 - 256}{9} = -2$

E.g. 2 The time taken for students aged 11 to run 100 m can be considered as having a normal distribution with a mean of 15.6 seconds and a standard deviation of 0.4 seconds. Find the probability that the time taken for a student to complete the race is:
(a) under 15 seconds
(b) at least 16 seconds
(c) between 15 and 16 seconds.
Give your answers to 4 s.f.

Working: (a) $X \sim \text{N}(15.6, 0.4^2)$
$P(X < 15) = 0.06681$

(b) $P(X \geq 16) = 0.1587$

(c) $P(15 < X < 16) = 0.7745$

E.g. 3 Scores on a test are normally distributed with a mean of 68 and a standard deviation of 8.

(a) Find the probability that a student scored:

(i) at least 75 on the test

(ii) at least 75 on the test given that the student scored at least 70 on the test.

(b) In a group of 50 students, how many students would you expect to score between 65 and 72 on the test. Give your answer to the nearest number of students.

Working: (a) (i) $X \sim \text{N}(68, 8^2)$

$P(X \geq 75) = 0.1908$

The probability a student scored at least 75 is 0.1908 (4 s.f.)

(ii) $$P(X \geq 75 \,|\, X \geq 70) = \frac{P(X \geq 75 \cap X \geq 70)}{P(X \geq 70)}$$
$$= \frac{P(X \geq 75)}{P(X \geq 70)}$$
$$\approx \frac{0.19079}{0.40129}$$
$$= 0.4754$$

The probability a student scored at least 75 given that they scored at least 70 is 0.4754 (4 s.f.)

(b) Number $= 50 \times P(65 < X < 72) = 50 \times 0.3376 = 16.9$

The number of students would you expect to score between 65 and 72 on the test is 17.

E.g. 4 If X is a normally distributed variable with a mean of 24 and a standard deviation of 2, find:

(a) $P(X > 28 \,|\, X > 26)$ (b) $P(26 < X < 28 \,|\, X > 27)$

Give your answers to 3 s.f..

Working: (a) $$P(X > 28 \,|\, X > 26) = \frac{P(X > 28 \cap X > 26)}{P(X > 26)}$$
$$= \frac{P(X > 28)}{P(X > 26)}$$
$$\approx \frac{0.02275}{0.15866}$$
$$= 0.143$$

(b) $$P(26 < X < 28 \,|\, X > 27) = \frac{P(26 < X < 28 \cap X > 27)}{P(X > 27)}$$
$$= \frac{P(27 < X < 28)}{P(X > 27)}$$
$$\approx \frac{0.044057}{0.066807}$$
$$= 0.659$$

Video: Standard normal distribution

Video: Probability using tables

Solutions to Starter and E.g.s

Exercise

p384 17B Qu 1i, 2i, 3-10, (11-14 red)